Holographic Equation of State

Maurício Hippert,
In collaboration with w/ J. Grefa, I. Portillo, J. Noronha,
J. Noronha-Hostler, C. Ratti and R. Rougemont

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Holographic EMD Model

- Model QGP via AdS_5 black-hole dual. Extra dimension: AdS radius r.

  P. Kovtun, D. T. Son, A. O. Starinets, PRL **94** (2005)
  S. S. Gubser and A. Nellore, PRD **78** (2008)

- "Black-hole engineering": can be matched to lattice results.

  R. Critelli, J. Noronha, J. Noronha-Hostler, I. Portillo, C. Ratti, R. Rougemont, PRD **96** (2017)
  J. Grefa, J. Noronha, J. Noronha-Hostler, I. Portillo, C. Ratti, R. Rougemont, PRD **104** (2021)

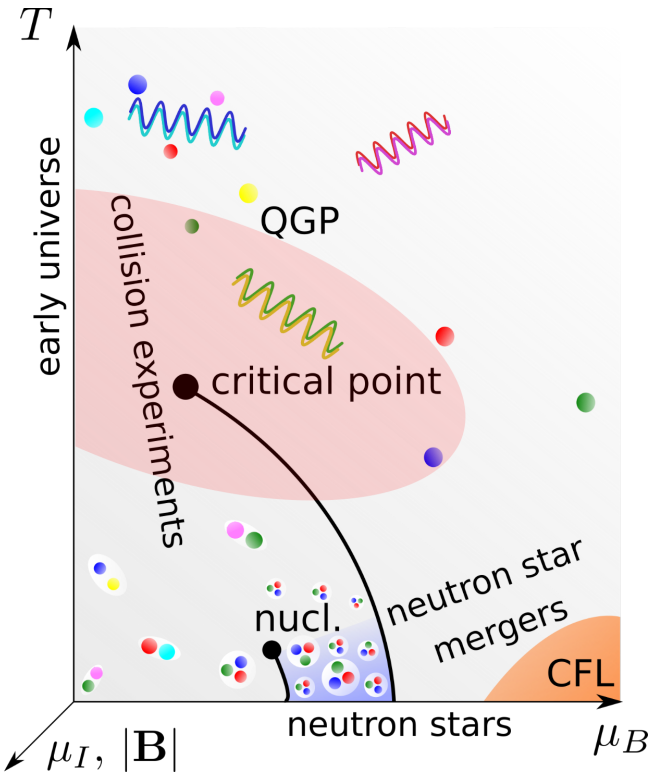
- Able to predict transport properties.

  S. S. Gubser, A. Nellore, S. S. Pufu and F. D. Rocha, PRL **101**, (2008)

- Can describe QCD phase transition.

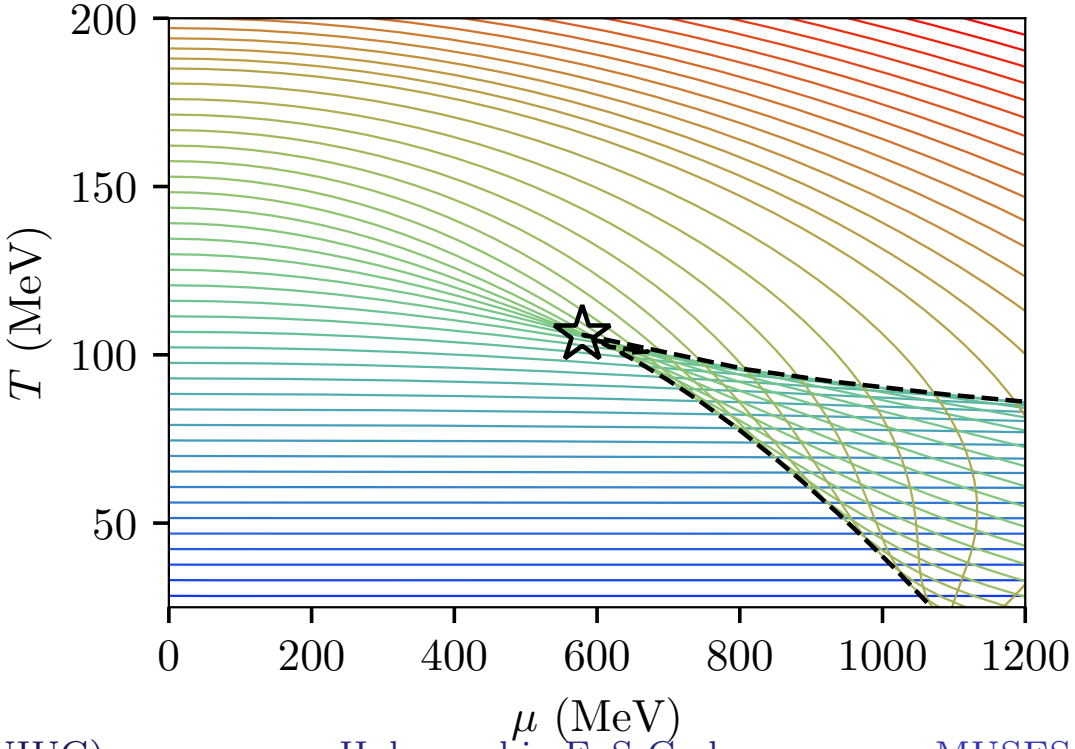
  O. DeWolfe, S. S. Gubser and C. Rosen, PRD **83** (2011)

- No asymptotic freedom or hadrons.

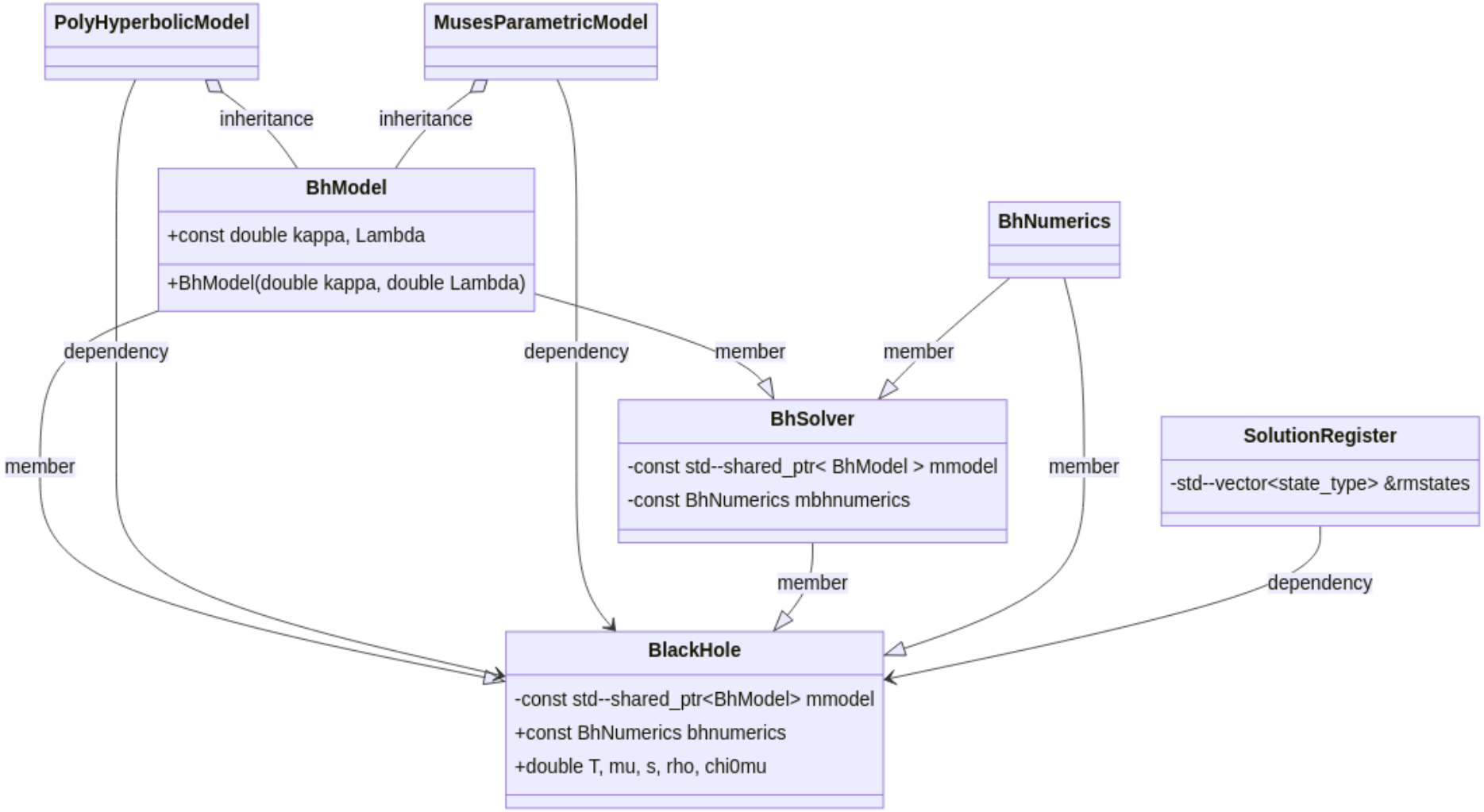


C++ implementation

1. Determine initial values $\phi = \phi_0$, $\frac{\partial A_0}{\partial r} = \Phi_1$ at $r = 0$.
2. Solution to EOM on grid in (ϕ_0, Φ_1).
3. Extract thermodynamics.
4. Interpolate grid in (T, μ).



C++ Module Code



Functionalities

- Choice of parametrization and predefined models. Abstract `BhModel` class.
- Equation of state for stable, metastable, and unstable phases. Maxwell construction.
- Finds transition and spinodal lines, critical point.
- Option to output full dependence on holographic radius.

Underway

- Common library `muses_yaml`: multi-dimensional table with direct output to `yaml` and `csv`.
- Integration to Calculation Engine.

Bayesian analysis

- Uncertainties on parameters and predictions.
- Unknown correlation for lattice errobars $\rightarrow$ extra parameter Γ.

Differential Evolution MC

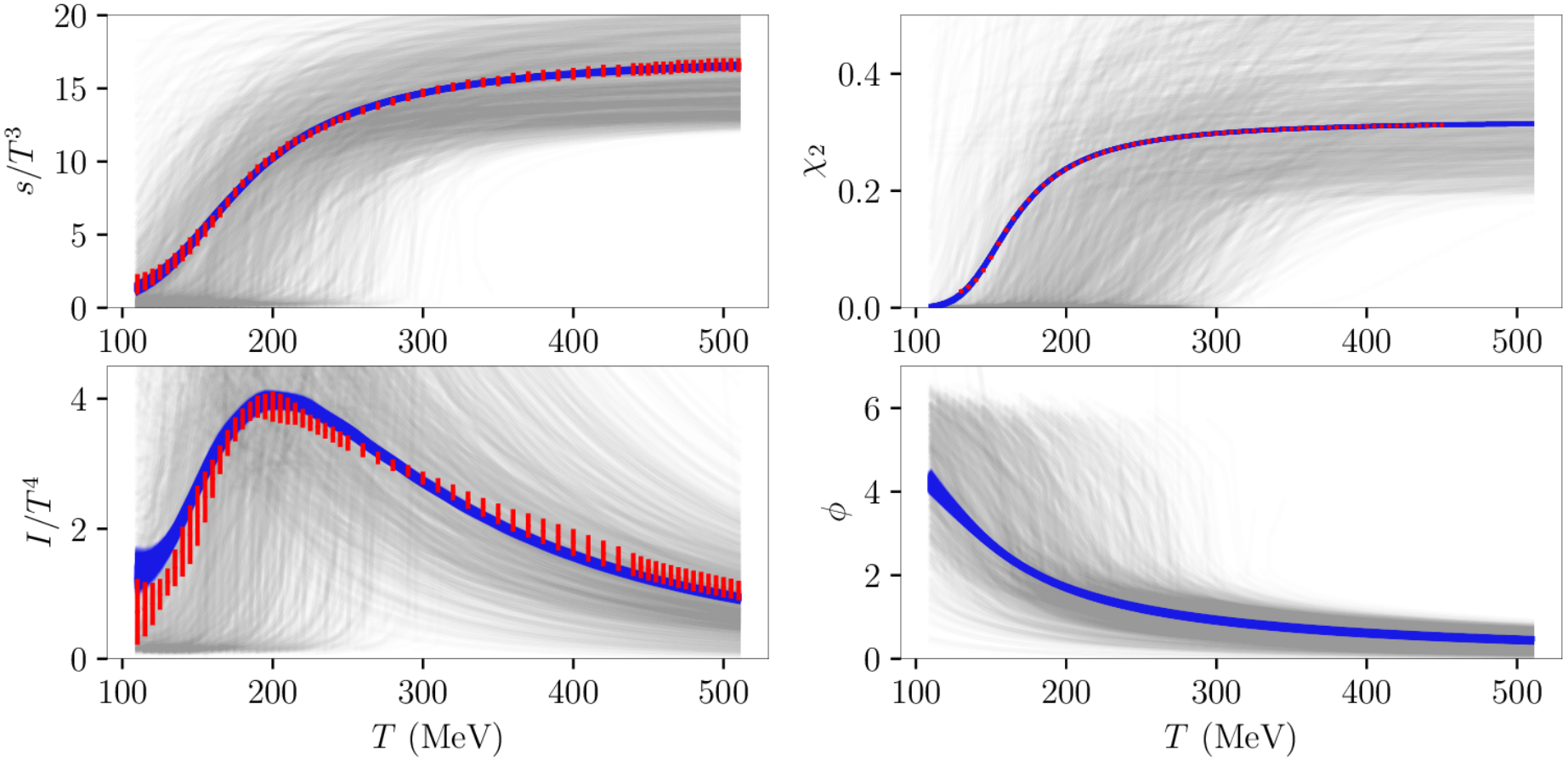
1. Use two MC chains to get step for a third one.
2. Compute $\mathcal{P}$ from model EoS.
   - If $\mathcal{P}/\mathcal{P}_0 > 1$, transition to new parameters.
   - Otherwise, accept transition with probability $\mathcal{P}/\mathcal{P}_0$.
3. Repeat.

Ter Braak, C. J., Statistics and Computing, **16** (2006)

Inputs: Baryon susceptibility and entropy density from the lattice.

S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg and K. K. Szabo, PRL **730** (2014)
Borsányi, Fodor, Guenther et al., PRL **126** (2021)

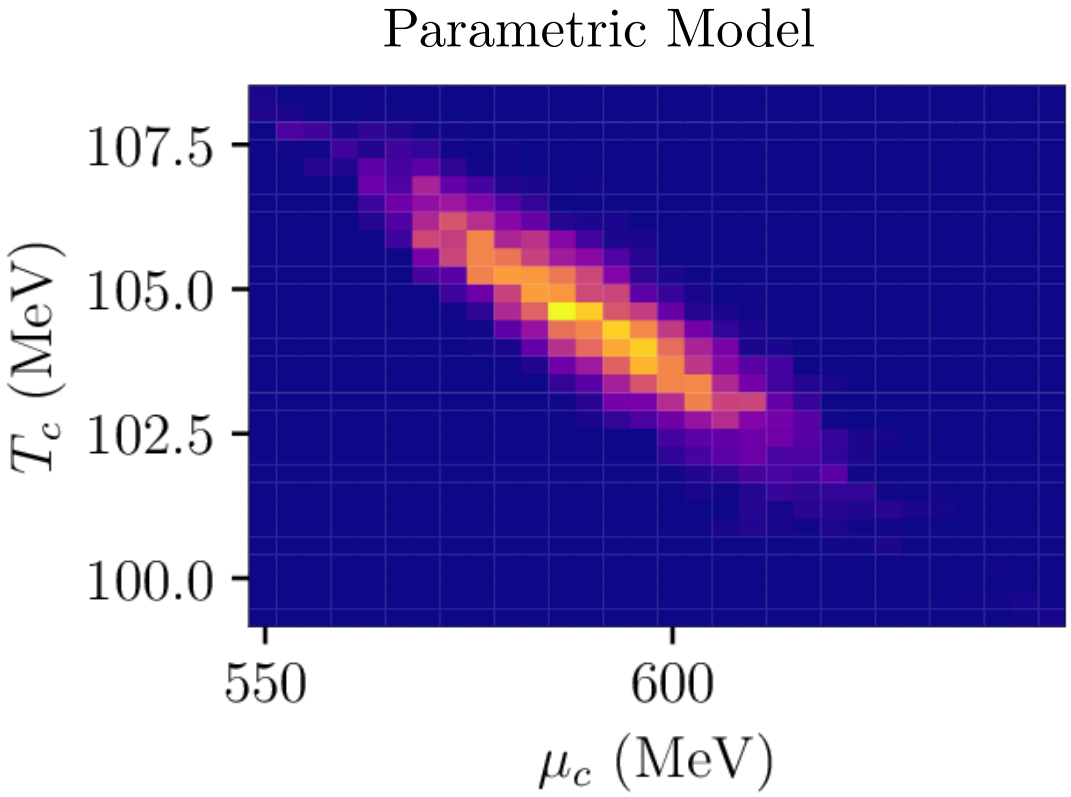
Posterior distribution: Equation of State



- Very tight constraints on entropy density and baryon susceptibility.

Bellwied, Borsanyi, Fodor et al., PRD **92** (2015)
Borsányi, Fodor, Guenther et al., PRL **126** (2021)

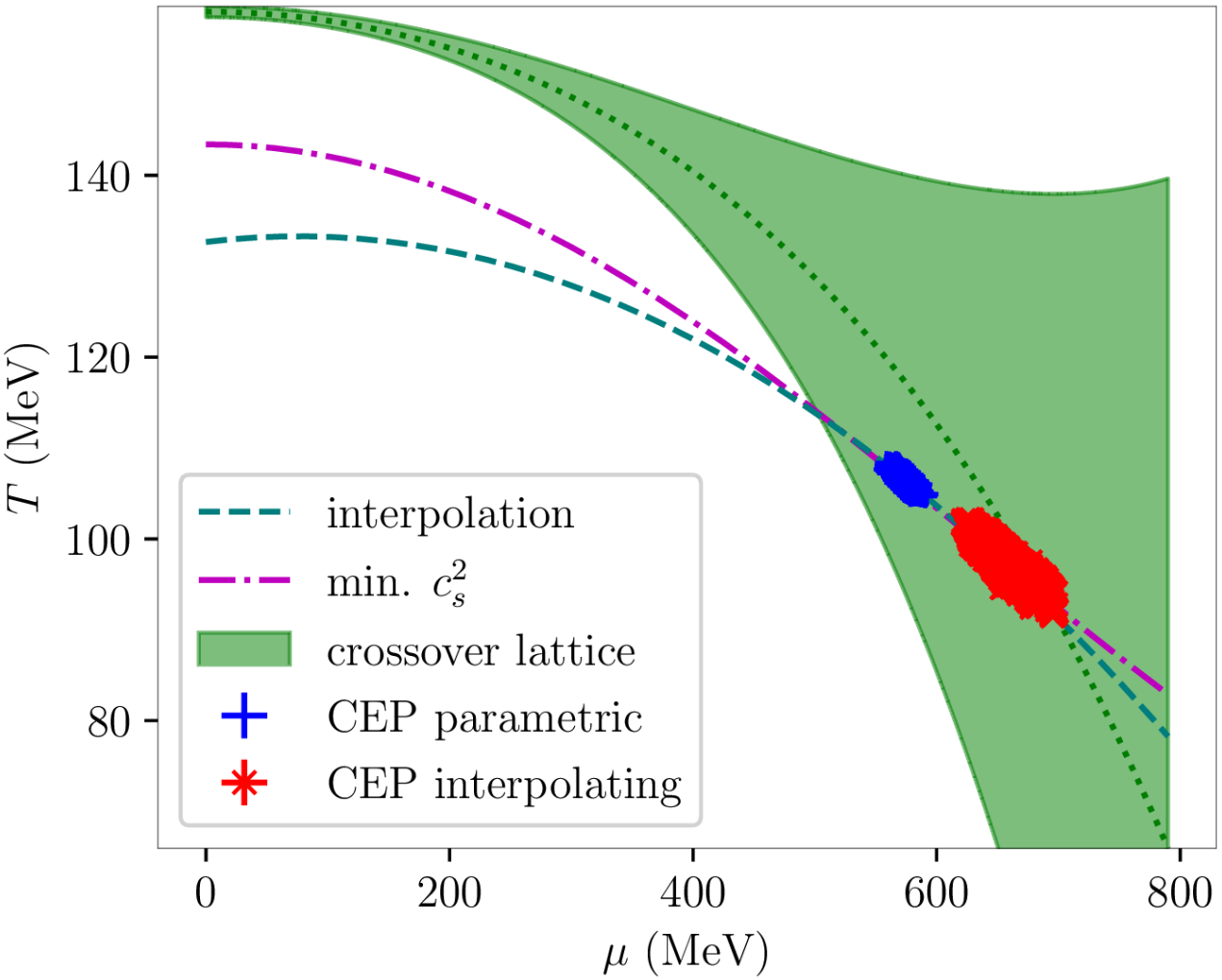
Finding the critical point



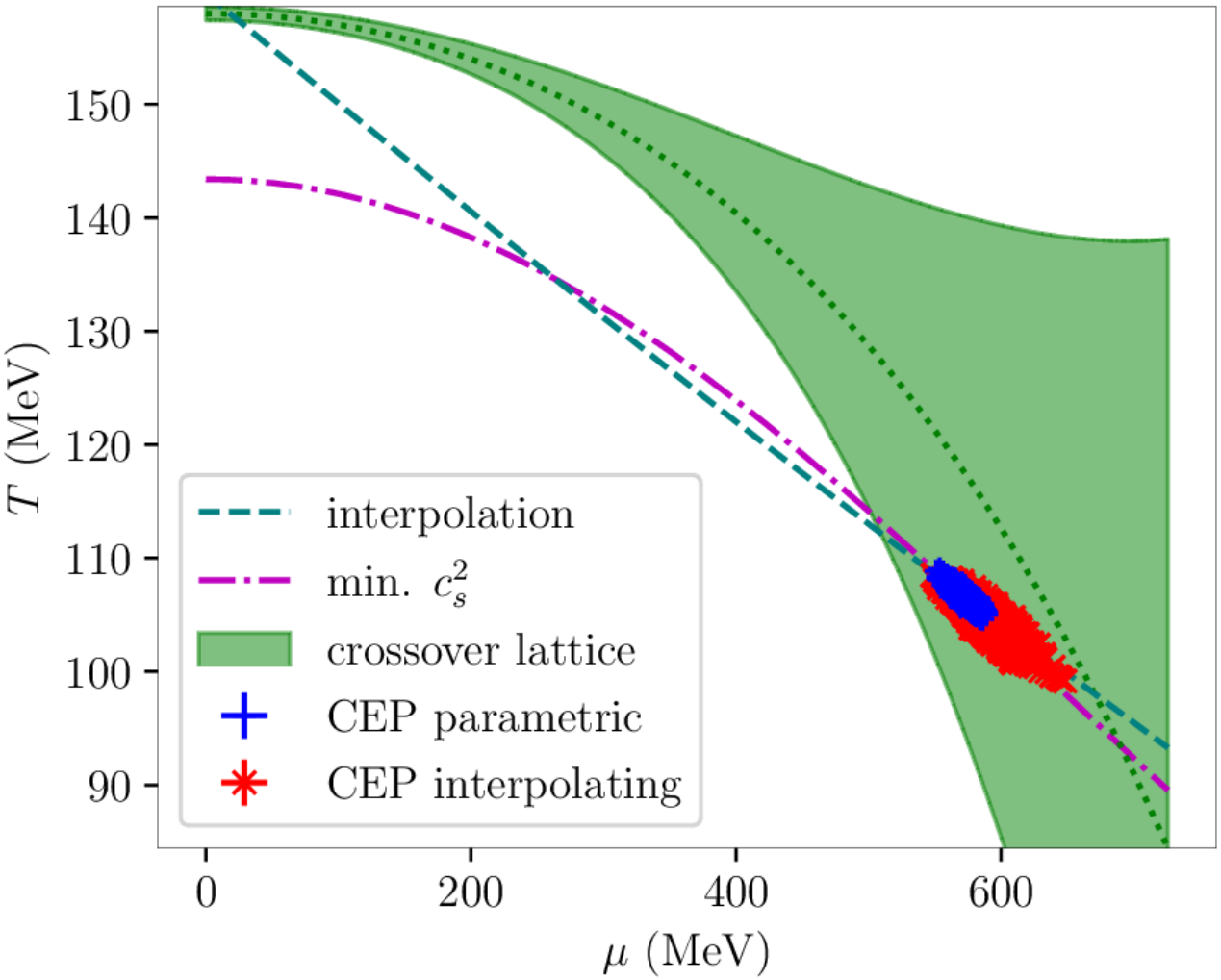
- Critical point not present in all prior samples.
- Estimate statistical preference for a critical point.

MH, J. Grefa, J. Noronha, J. Noronha-Ho stler, I. Portillo, C. Ratti, R. Rougemont, to appear.

Bayesian


140
120
100
80
T (MeV)
0
200
400
600
800
μ (MeV)
interpolation
min. c_s^2
crossover lattice
CEP parametric
CEP interpolating

Bayesian improved



Challenges

1. Good choice of initial grid in (ϕ_0, Φ_1). Solution by finding maximum Φ_1 for each ϕ_0. Set steps in ϕ_0 to equal spacing of T at $\mu = 0$.
2. Interpolating grid with small ΔT, $\Delta\mu \to$ division by small numbers $\to$ noisy results.
3. Bayesian analysis very slow to converge. Solution via tempering and enforcing correct distribution when changing the temperature.
4. Slow I/O in `yaml` format when computing EoS on a fine grid.

Current status

Achieved

1. Implementation in C++ of full equation of state.
2. Parallelization with `OpenMP`.
3. Bayesian improved to include correlation between lattice points and to reduce auto-correlation time.

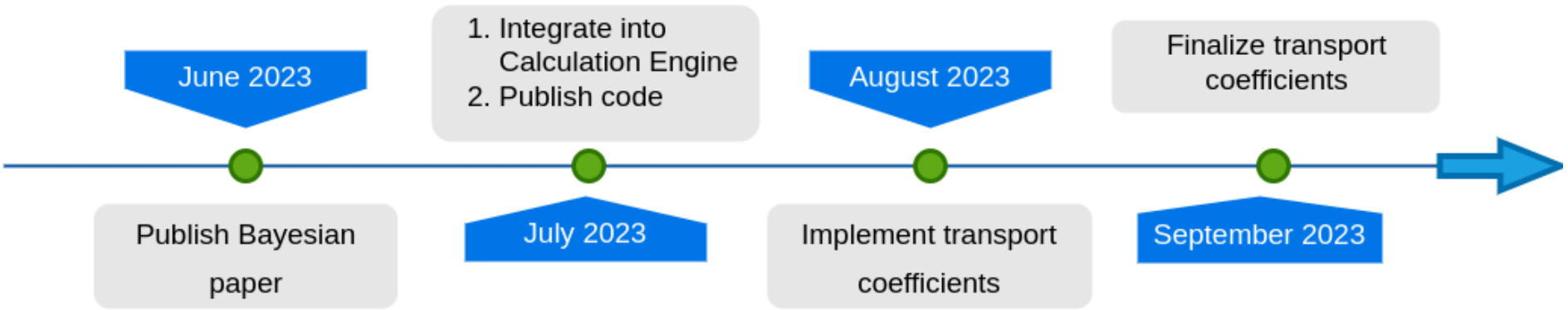
In progress

1. Improvements on code readability and documentation.
2. Common `muses_yaml` library for output.

To do

1. Integrate with calculation engine.
2. Implement transport coefficients.
3. Extension to include isospin and strangeness.

Timeline


June 2023
Publish Bayesian paper
1. Integrate into Calculation Engine
2. Publish code
July 2023
August 2023
Implement transport coefficients
Finalize transport coefficients
September 2023

Isospin and Strangeness

- Extension to include isospin and strangeness expected.
- Candidate action:

$$\mathcal{S} = \frac{1}{\kappa_5^2}\int_{\mathcal{M}_5} dx^5\sqrt{-g}\left\{R - \frac{1}{2}(\partial_\mu\phi)^2 - V(\phi) - \frac{f(\phi)}{4}F^{\mu\nu}F_{\mu\nu} + \right.$$
$$\left. +h(\phi)\left[\mathrm{tr}\,|D_\mu X|^2 - V(X)\right] - \frac{g(\phi)}{4}\,\mathrm{tr}\left[G^{\mu\nu}_{(L)}G^{(L)}_{\mu\nu} + G^{\mu\nu}_{(R)}G^{(R)}_{\mu\nu}\right]\right\}$$

- Flavor and chiral symmetries: non-Abelian $SU(3)$ gauge fields $G^{(L,R)}_{\mu\nu}$ in the bulk.
- Higgs field X for spontaneous and explicit symmetry breakings.
- Equations of motion in equilibrium and numerical implementation missing. Task for next year?

Conclusions

1. Model description of strongly-coupled QGP.
2. Holographic EOS migrated to C++.
3. Systematic scan of parameter space.

Outlook

1. Publication of Bayesian analysis: `June/2023`
2. Integration to Calculation Engine and public code: `July/2023`
3. Transport coefficients: `August - September/2023`
4. Extension to include isospin and strangeness chemical potentials.

Backup slides...

Einstein-Maxwell-Dilaton model

- Breaking of conformal symmetry: dilaton field ϕ.
- Dual to baryon chemical potential μ: Abelian gauge field A^μ.
- Action:

$$S = \frac{1}{2\kappa_5^2} \int_{\mathcal{M}_5} d^5x \sqrt{-g} \left[R - \frac{(\partial_\mu \phi)^2}{2} - V(\phi) - \frac{f(\phi) F_{\mu\nu}^2}{4} \right],$$

- Two potentials, $V(\phi)$ and $f(\phi)$, tweaked to fit lattice QCD results.

S. S. Gubser and A. Nellore, PRD **78** (2008)
O. DeWolfe, S. S. Gubser and C. Rosen, PRD **83** (2011)
R. Critelli, J. Noronha, J. Noronha-Hostler, I. Portillo, C. Ratti, R. Rougemont, PRD **96** (2017)
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Phenomenological holographic potentials

Polynomial-Hyperbolic Parametrization

- Interpolates between `arXiv:1706.00455` and `arXiv:2201.02004`

$$V(\phi) = -12\cosh(\gamma\,\phi) + b_2\,\phi^2 + b_4\,\phi^4 + b_6\,\phi^6$$

$$f(\phi) = \frac{\mathrm{sech}(c_1\phi + c_2\phi^2 + c_3\phi^3)}{1+d_1} + \frac{d_1}{1+d_1}\,\mathrm{sech}(d_2\phi)$$

Parametric Approach

- Similar shapes, more interpretable parameters

$$V(\phi) = -12\cosh\left[\left(\frac{\gamma_1\,\Delta\phi_V^2 + \gamma_2\,\phi^2}{\Delta\phi_V^2 + \phi^2}\right)\phi\right]$$

$$f(\phi) = 1-(1-A_1)\left[\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{\phi-\phi_1}{\delta\phi_1}\right)\right] - A_1\left[\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{\phi-\phi_2}{\delta\phi_2}\right)\right]$$

Equations of motion

$$\phi''(r) + \left[\frac{h'(r)}{h(r)} + 4A'(r) - B'(r)\right]\phi'(r) - \frac{e^{2B(r)}}{h(r)}\left[\frac{\partial V(\phi)}{\partial\phi} + \right.$$
$$\left. - \frac{e^{-2[A(r)+B(r)]}\Phi'(r)^2}{2}\frac{\partial f(\phi)}{\partial\phi}\right] = 0,$$

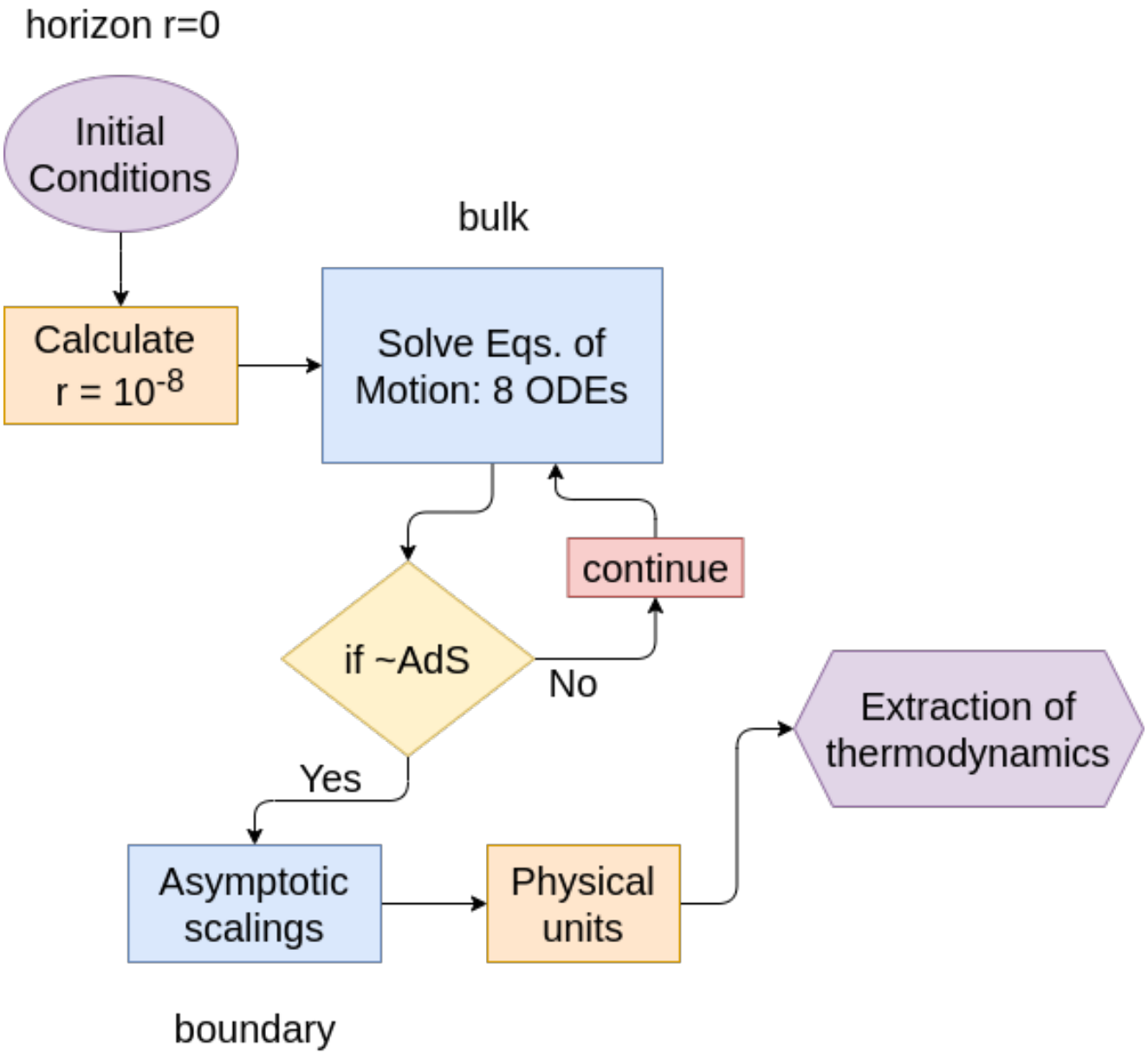
$$\Phi''(r) + \left[2A'(r) - B'(r) + \frac{d[\ln f(\phi)]}{d\phi}\phi'(r)\right]\Phi'(r) = 0,$$

$$A''(r) - A'(r)B'(r) + \frac{\phi'(r)^2}{6} = 0,$$

$$h''(r) + [4A'(r) - B'(r)]h'(r) - e^{-2A(r)}f(\phi)\Phi'(r)^2 = 0,$$

$$h(r)[24A'(r)^2 - \phi'(r)^2] + 6A'(r)h'(r) +$$
$$+2e^{2B(r)}V(\phi) + e^{-2A(r)}f(\phi)\Phi'(r)^2 = 0,$$

Black-Hole Engineering: Practice



Relaxational method

- Alternatively, add new (relaxation) equation:

$$\frac{dC}{dr} = -\Gamma_C \left(C - \phi\, e^{\nu A}\right)$$

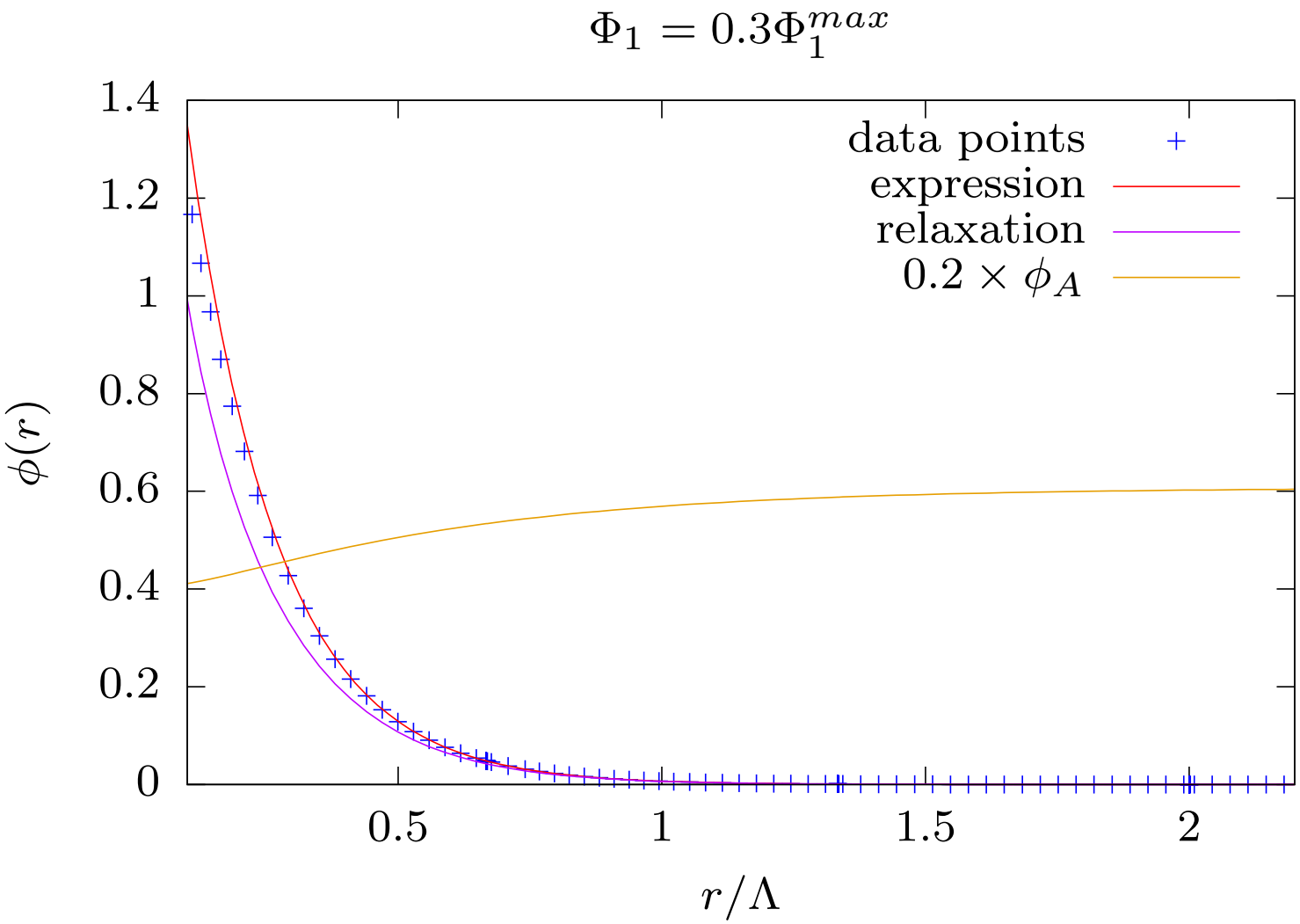
1. Fixed point $C \sim \phi\, e^{\nu A} \to \phi_A$ as $r \to \infty$
2. Convergence if $|C - \phi\, e^{\nu A}|/C \ll 1$.
3. Simultaneous to EOM. No need to even store $\phi(r)$

- Check by determining Φ_2^{far} from

$$\frac{dD}{dr} = -\Gamma_D \left(D - \Phi'\, e^{2A}\right)$$

- Choice: $\Gamma_D = \Gamma_C = 2$

Relaxational method – $T = 162$ MeV, $\mu = 543$ MeV



Validation of relaxational method

- Comparing Φ_2^{far} from relaxation and conserved quantities:
  1. 5×10^{-5} precision for $T = 196$ MeV, $\mu = 736$ MeV
  2. 9×10^{-5} precision for $T = 162$ MeV, $\mu = 543$ MeV
  3. 0.003 precision for $T = 27$ MeV, $\mu = 4861$ MeV
  4. 0.004 precision for $T = 68$ MeV, $\mu = 471$ MeV
- Comparing with ϕ_A from previous codes:

ϕ_0	$\frac{\Phi_1}{\Phi_1^{\max}}$	RR	JG	C++
2	0	2.9481	2.9458	2.9482
2	0.3	3.03802	3.0414	3.03797
5	0	28.1668	28.1336	28.1648
5	0.3	34.5442	34.5627	34.5459

- With $\Gamma_C \to 10 \times \Gamma_C$, last point moves to 34.5438...

Thermodynamics

$$T = \frac{1}{4\pi\phi_A^{1/\nu}\sqrt{h_0^{\text{far}}}}\Lambda,$$

$$\mu_B = \frac{\Phi_0^{\text{far}}}{\phi_A^{1/\nu}\sqrt{h_0^{\text{far}}}}\Lambda,$$

$$s = \frac{2\pi}{\kappa_5^2\phi_A^{3/\nu}}\Lambda^3,$$

$$\rho_B = -\frac{\Phi_2^{\text{far}}}{\kappa_5^2\phi_A^{3/\nu}\sqrt{h_0^{\text{far}}}}\Lambda^3.$$