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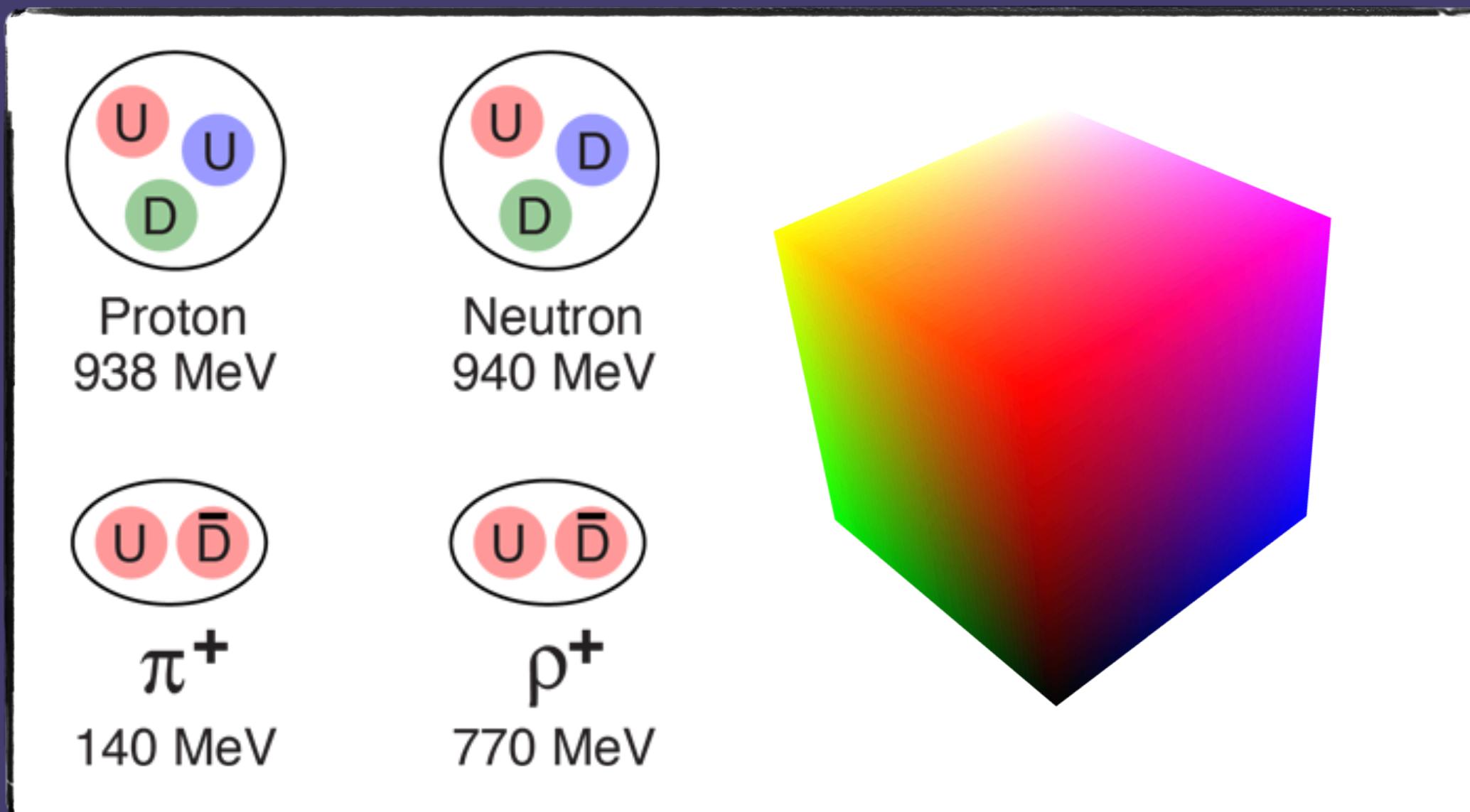
in collaboration with Alejandro Florez,
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Contribution of Hadron Families to the QCD Equation of State

Quantum Chromodynamics

Quantum Chromodynamics (QCD) is the gauge field theory which describes the interactions between quarks and gluons. It is the richest gauge theory we know of exhibiting interesting emergent phenomena, color structure, non-perturbative dynamics, etc. all encoded in the concise form of the QCD Lagrangian.



QCD LAGRANGIAN

$$\mathcal{L} = \bar{\psi}_q^i (i\gamma^\mu) (D_\mu)_{ij} \psi_q^j - m_q \bar{\psi}_q^i \psi_{qi} - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}$$

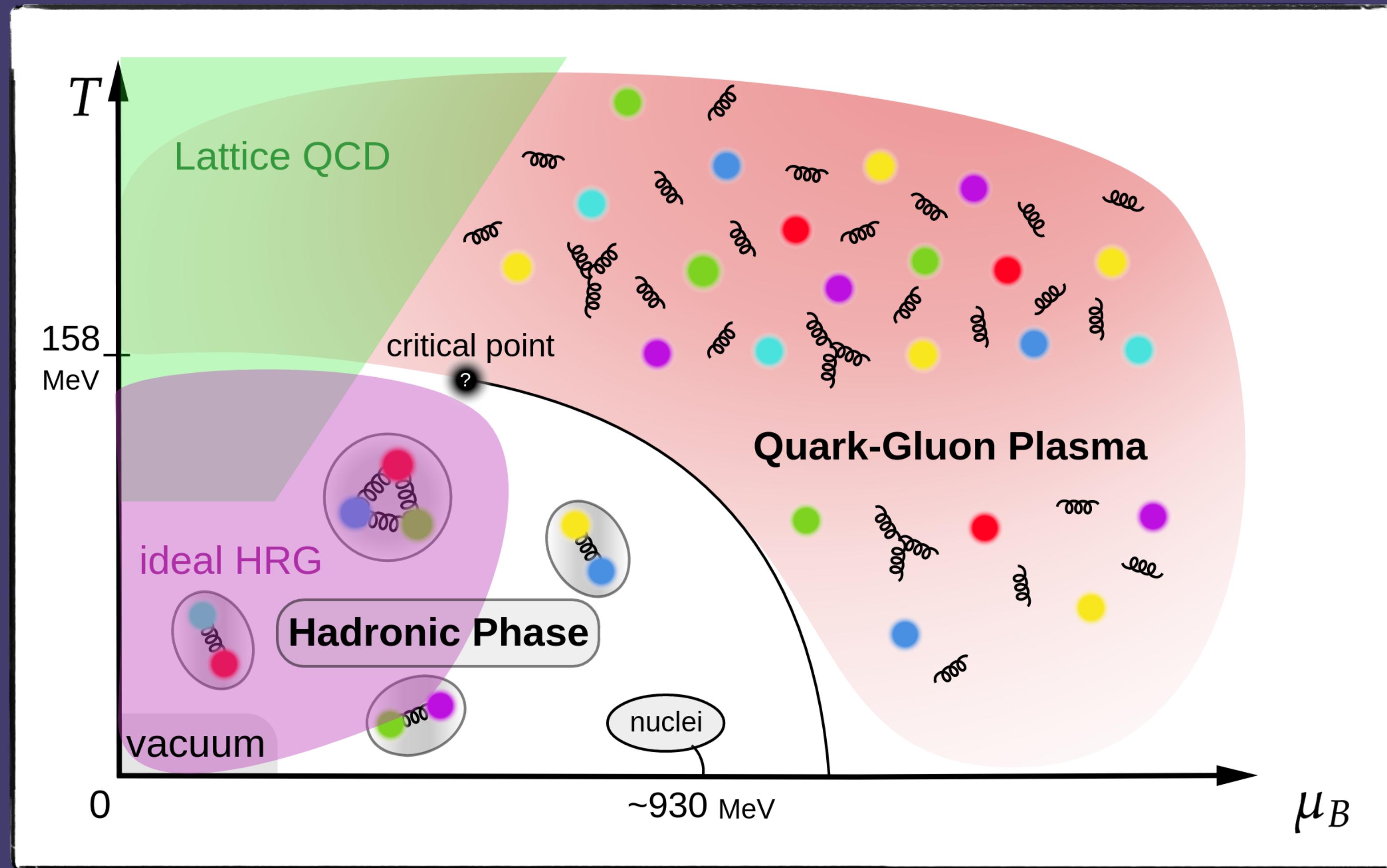
$$\psi_q = (\psi_{qR}, \psi_{qG}, \psi_{qB})^T$$

$$(D_\mu)_{ij} = \delta_{ij} \partial_\mu - i g_s t_{ij}^a A_\mu^a$$

$$F_{\mu\nu}^a = \underbrace{\partial_\mu A_\nu^a - \partial_\nu A_\mu^a}_{\text{Abelian}} + \underbrace{g_s f^{abc} A_\mu^b A_\nu^c}_{\text{non-Abelian}}$$

Quarks and gluons form color-neutral composite particles called hadrons. Three different colored quarks form baryons or color/anti-color combinations form mesons

Lattice QCD and Hadron Resonance Gas



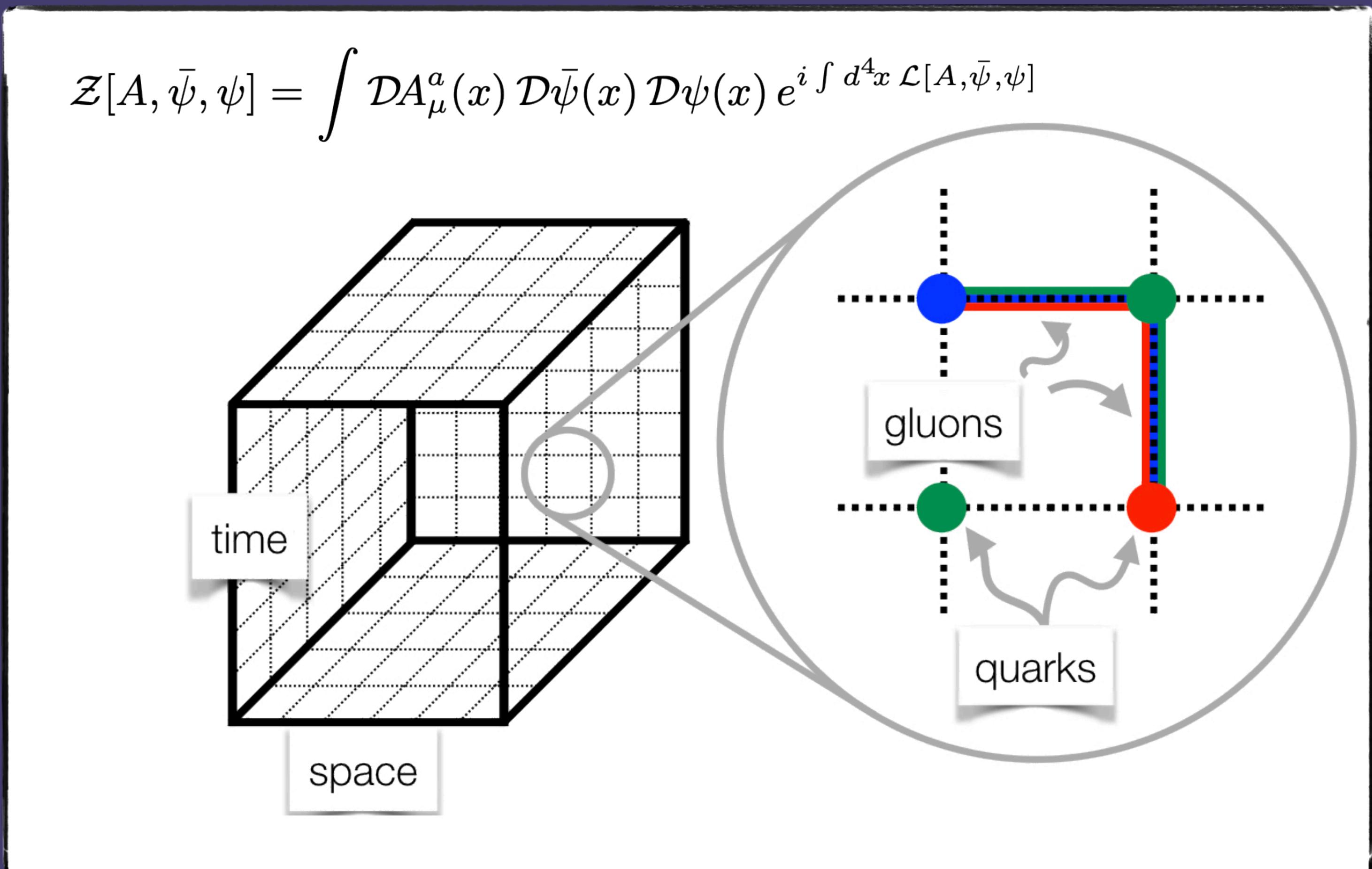
Lattice QCD

Lattice QCD is a non-perturbative approach to solving QCD on a four-dimensional lattice where:

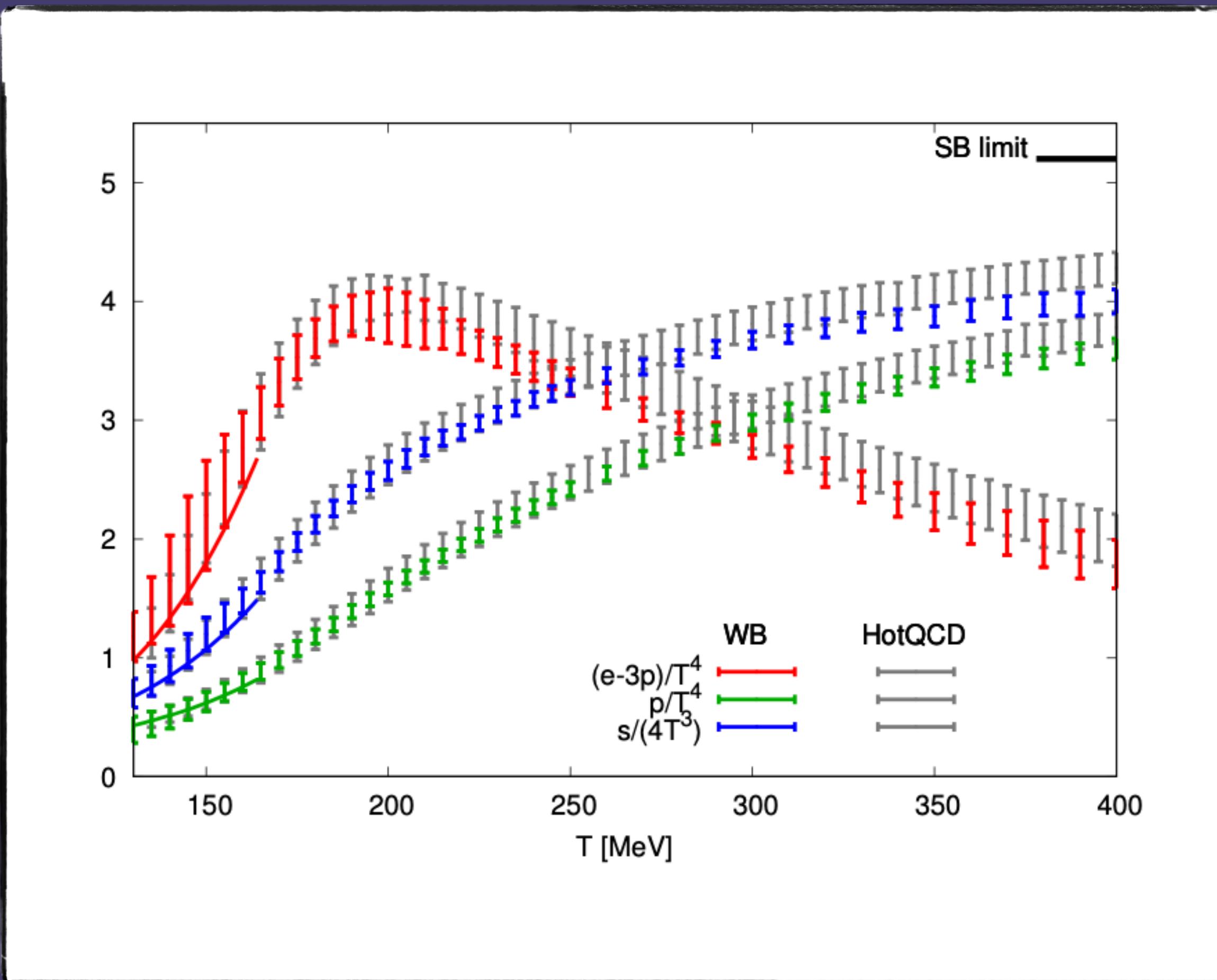
Quark fields are defined on lattice sites

Gluon fields are defined at the links

Monte Carlo Path Integrals are then used to find solutions:



Lattice QCD Equation of State



Lattice QCD produces thermodynamic observables

The region described by the Hadron Resonance Gas model
(Low T) agrees with Lattice

How do individual Hadron families contribute to the
full pressure?

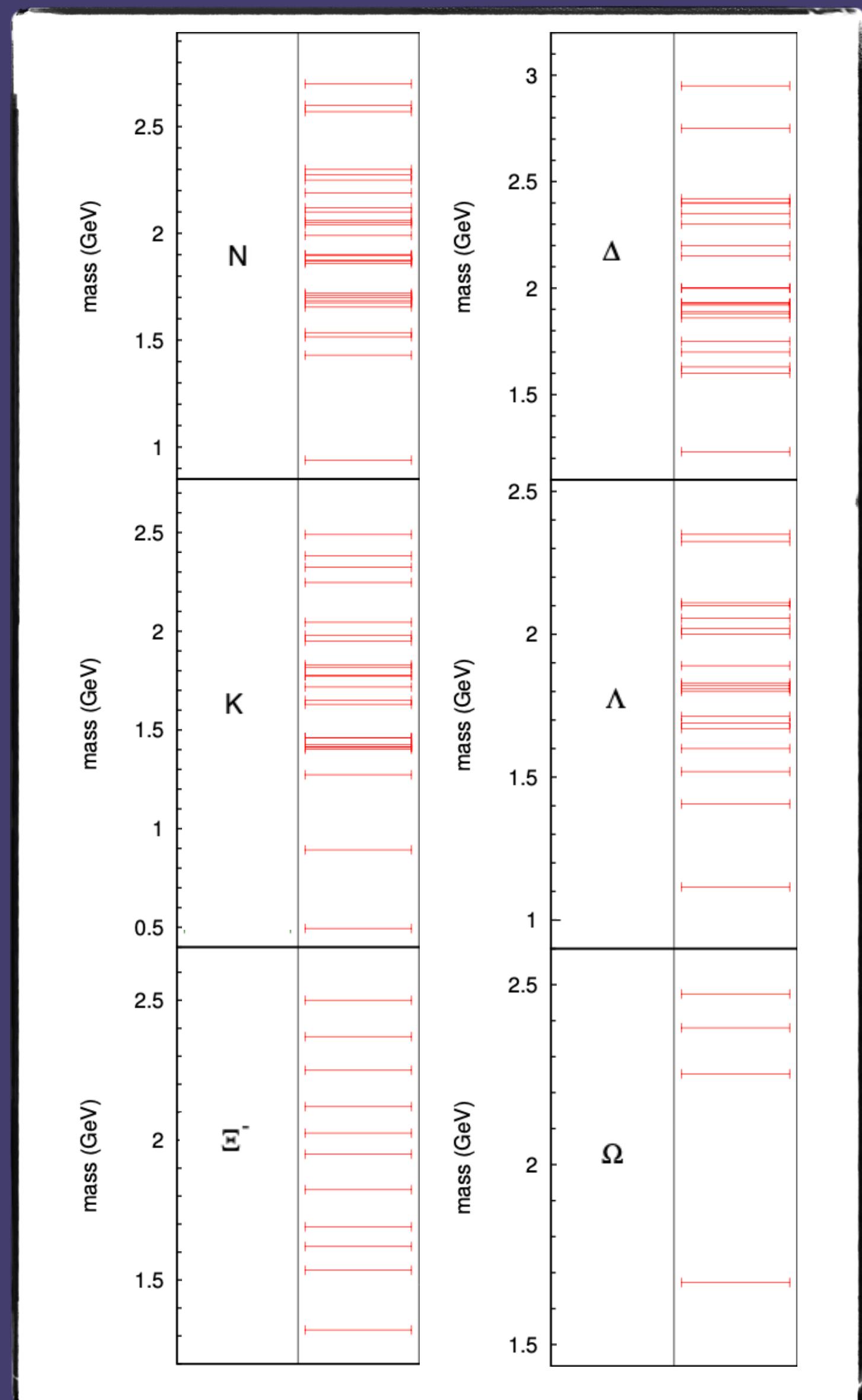
Hadron Resonance Gas (HRG) Model

$$p(T, \mu_B, \mu_Q, \mu_S) = \sum_{i \in \text{HRG}} (-1)^{B_i+1} \frac{d_i T}{(2\pi)^3} \int d^3 \vec{p} \ln [1 + (-1)^{B_i+1} \exp \{-(\sqrt{\vec{p}^2 + m_i^2} - B_i \mu_B - S_i \mu_S - Q_i \mu_Q)/T\}]$$

Model introduced by Hagedorn which describes a gas of non-interacting hadrons in their ground state.

Can be summed over **ALL KNOWN** hadronic states for the full pressure or **HADRON FAMILIES** for partial pressures

HRG model allows us to identify the contribution of a given hadron family to the thermodynamics, identified by the values of **baryon number, electric charge and strangeness**.



Families of Hadrons

$$\begin{aligned} \ln Z^{HRG}(T, V, \mu) &= \sum_{i \in PDG} \ln Z_i(T, V, \mu) = \\ &= \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln [1 \pm \lambda_i(T, \mu) \exp(-\beta \epsilon_i)] \\ \lambda_i(T, \mu) &= \exp \left[\frac{B_i \mu_B + S_i \mu_S + Q_i \mu_Q}{T} \right] \end{aligned}$$

Grouped by Quantum Numbers:
B = Baryon Number
S = Strangeness
Q = Electric Charge

Boltzmann Approximation

Making use of the fugacity expansion

$$\frac{p_B}{T^4} = \sum_{i \in baryons} \frac{g_i}{\pi^2} \left(\frac{m_i}{T} \right)^2 \sum_{N=1}^{\infty} (-1)^{N+1} N^{-2} K_2(N \frac{m_i}{T}) \cosh \left[N \frac{\mu_B}{T} \right]$$

Susceptibilities

$$\chi_{mnk}^{BSQ} = \left. \frac{\partial^{(m+n+k)} [p(\hat{\mu}_B, \hat{\mu}_S, \hat{\mu}_Q)/T^4]}{\partial \hat{\mu}_B^m \partial \hat{\mu}_S^n \partial \hat{\mu}_Q^k} \right|_{\vec{\mu}=0}$$

Keeping the first term (valid for large arguments)

$$\frac{p_B}{T^4} = \sum_{i \in baryons} \frac{g_i}{\pi^2} \left(\frac{m_i}{T} \right)^2 K_2 \left(\frac{m_i}{T} \right) \cosh \left(\frac{\mu_B}{T} \right) = F(T) \cosh \left(\frac{\mu_B}{T} \right)$$

Useful Behavior

$$\frac{\partial^n (p/T^4)}{\partial (\mu_B/T)^n} = F(T) \cosh \left(\frac{\mu_B}{T} \right) \quad \text{for } n \text{ even,}$$

$$\frac{\partial^n (p/T^4)}{\partial (\mu_B/T)^n} = F(T) \sinh \left(\frac{\mu_B}{T} \right) \quad \text{for } n \text{ odd.}$$

Full Pressure

$$\begin{aligned}
 P(\hat{\mu}_B, \hat{\mu}_S, \hat{\mu}_Q) = & P_{000} + \underline{P_{00|1|}} \cosh(\hat{\mu}_Q) + \underline{P_{100}} \cosh(\hat{\mu}_B) \\
 & + \underline{P_{101}} \cosh(\hat{\mu}_B + \hat{\mu}_Q) + \underline{P_{10-1}} \cosh(\hat{\mu}_B - \hat{\mu}_Q) \\
 & + \underline{P_{102}} \cosh(\hat{\mu}_B + 2\hat{\mu}_Q) + \underline{P_{0|1|0}} \cosh(\hat{\mu}_S) \\
 & + \underline{P_{0|1||1|}} \cosh(\hat{\mu}_S + \hat{\mu}_Q) + \underline{P_{1|1|0}} \cosh(\hat{\mu}_B - \hat{\mu}_S) \\
 & + \underline{P_{1|1|1}} \cosh(\hat{\mu}_B - \hat{\mu}_S + \hat{\mu}_Q) \\
 & + \underline{P_{1|1|-1}} \cosh(\hat{\mu}_B - \hat{\mu}_S - \hat{\mu}_Q) \\
 & + \underline{P_{1|2|0}} \cosh(\hat{\mu}_B - 2\hat{\mu}_S) \\
 & + \underline{P_{1|2||1|}} \cosh(\hat{\mu}_B - 2\hat{\mu}_S - \hat{\mu}_Q) \\
 & + \underline{P_{1|3||1|}} \cosh(\hat{\mu}_B - 3\hat{\mu}_S - \hat{\mu}_Q).
 \end{aligned}$$

$$\chi_{mnk}^{BSQ} = \left. \frac{\partial^{(m+n+k)} [p(\hat{\mu}_B, \hat{\mu}_S, \hat{\mu}_Q)/T^4]}{\partial \hat{\mu}_B^m \partial \hat{\mu}_S^n \partial \hat{\mu}_Q^k} \right|_{\vec{\mu}=0}$$

B = Baryon Number
S = Strangeness
Q = Electric Charge

$$P_{1|2|0} \cdots \frac{\partial^2 P_{1|2|0} \cosh(\hat{\mu}_B - 2\hat{\mu}_S)}{\partial \hat{\mu}_s^2} = (-2)^2 * P_{1|2|0} \cosh(\hat{\mu}_B - 2\hat{\mu}_S)$$

Susceptibilities

$$\begin{aligned}
 P(\hat{\mu}_B, \hat{\mu}_S, \hat{\mu}_Q) = & P_{000} + P_{00|1|} \cosh(\hat{\mu}_Q) + P_{100} \cosh(\hat{\mu}_B) \\
 & + P_{101} \cosh(\hat{\mu}_B + \hat{\mu}_Q) + P_{10-1} \cosh(\hat{\mu}_B - \hat{\mu}_Q) \\
 & + P_{102} \cosh(\hat{\mu}_B + 2\hat{\mu}_Q) + P_{0|1|0} \cosh(\hat{\mu}_S) \\
 & + P_{0|1||1|} \cosh(\hat{\mu}_S + \hat{\mu}_Q) + P_{1|1|0} \cosh(\hat{\mu}_B - \hat{\mu}_S) \\
 & + P_{1|1|1} \cosh(\hat{\mu}_B - \hat{\mu}_S + \hat{\mu}_Q) \\
 & + P_{1|1|-1} \cosh(\hat{\mu}_B - \hat{\mu}_S - \hat{\mu}_Q) \\
 & + P_{1|2|0} \cosh(\hat{\mu}_B - 2\hat{\mu}_S) \\
 & + P_{1|2||1|} \cosh(\hat{\mu}_B - 2\hat{\mu}_S - \hat{\mu}_Q) \\
 & + P_{1|3||1|} \cosh(\hat{\mu}_B - 3\hat{\mu}_S - \hat{\mu}_Q).
 \end{aligned}$$

 χ_{BSQ}

$$\begin{bmatrix} \chi_{200} \\ \chi_{002} \\ \chi_{020} \\ \chi_{101} \\ \chi_{110} \\ \chi_{011} \\ \chi_{400} \\ \chi_{004} \\ \chi_{040} \\ \chi_{301} \\ \chi_{310} \\ \chi_{013} \\ \chi_{103} \\ \chi_{130} \\ \chi_{031} \\ \chi_{202} \\ \chi_{220} \\ \chi_{022} \\ \chi_{211} \\ \chi_{112} \\ \chi_{121} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 4 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 4 & 4 & 9 \\ 0 & 0 & 1 & -1 & 2 & 0 & 0 & 0 & 1 & -1 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -2 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 1 & 0 & 2 & 3 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 16 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 16 & 16 & 81 \\ 0 & 0 & 1 & -1 & 2 & 0 & 0 & 0 & 1 & -1 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -2 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 1 & 0 & 2 & 3 \\ 0 & 0 & 1 & -1 & 8 & 0 & 0 & 0 & 1 & -1 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -8 & -8 & -27 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 1 & 0 & 8 & 27 \\ 0 & 0 & 1 & 1 & 4 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 4 & 4 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 4 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & -4 & -9 \end{bmatrix}$$

 P_{BSQ}

$$\begin{bmatrix} P_{00000} \\ P_{1000} \\ P_{101} \\ P_{10-1} \\ P_{102} \\ P_{01110} \\ P_{011111} \\ P_{11110} \\ P_{11111} \\ P_{1111-1} \\ P_{11210} \\ P_{112111} \\ P_{1131111} \end{bmatrix}$$

Susceptibilities

$$\begin{aligned}
P(\hat{\mu}_B, \hat{\mu}_S, \hat{\mu}_Q) = & P_{000} + \cancel{P_{00|1|}} \cosh(\hat{\mu}_Q) + \cancel{P_{100}} \cosh(\hat{\mu}_B) \\
& + \cancel{P_{101}} \cosh(\hat{\mu}_B + \hat{\mu}_Q) + \cancel{P_{10-1}} \cosh(\hat{\mu}_B - \hat{\mu}_Q) \\
& + \cancel{P_{102}} \cosh(\hat{\mu}_B + 2\hat{\mu}_Q) + \cancel{P_{0|1|0}} \cosh(\hat{\mu}_S) \\
& + \cancel{P_{0|1||1|}} \cosh(\hat{\mu}_S + \hat{\mu}_Q) + \cancel{P_{1|1|0}} \cosh(\hat{\mu}_B - \hat{\mu}_S) \\
& + \cancel{P_{1|1|1}} \cosh(\hat{\mu}_B - \hat{\mu}_S + \hat{\mu}_Q) \\
& + \cancel{P_{1|1|-1}} \cosh(\hat{\mu}_B - \hat{\mu}_S - \hat{\mu}_Q) \\
& + \cancel{P_{1|2|0}} \cosh(\hat{\mu}_B - 2\hat{\mu}_S) \\
& + \cancel{P_{1|2||1|}} \cosh(\hat{\mu}_B - 2\hat{\mu}_S - \hat{\mu}_Q) \\
& + \cancel{P_{1|3||1|}} \cosh(\hat{\mu}_B - 3\hat{\mu}_S - \hat{\mu}_Q).
\end{aligned}$$

$$P_{102} = -\frac{1}{6}\chi_{31}^{BQ} + \frac{1}{6}\chi_{13}^{BQ}$$

χ_{B5}

$$\begin{bmatrix}
 \chi_{200} \\
 \chi_{002} \\
 \chi_{020} \\
 \chi_{101} \\
 \chi_{110} \\
 \chi_{011} \\
 \chi_{400} \\
 \chi_{004} \\
 \chi_{040} \\
 \chi_{301} \\
 \chi_{310}
 \end{bmatrix}
 = \begin{bmatrix}
 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
 1 & 0 & 1 & 1 & 4 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\
 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 4 & 4 & 9 \\
 0 & 0 & 1 & -1 & 2 & 0 & 0 & 0 & 1 & -1 & 0 & -1 & -1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -2 & -2 & -3 \\
 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 1 & 0 & 2 & 3 \\
 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
 1 & 0 & 1 & 1 & 16 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\
 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 16 & 16 & 81 \\
 0 & 0 & 1 & -1 & 2 & 0 & 0 & 0 & 1 & -1 & 0 & -1 & -1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -2 & -2 & -3
 \end{bmatrix}$$

$$\begin{array}{ccccccccccccc} 0 & | & -1 & 2 & 0 & 0 & 0 & | & -1 & 0 & -1 & -1 \\ 0 & | & -1 & 8 & 0 & 0 & 0 & | & -1 & 0 & -1 & -1 \end{array}$$

χ_{220}		0 0 0 0 0 0 0 1 1 1 4 4 9
χ_{022}		0 0 0 0 0 0 1 0 1 1 0 4 9
χ_{211}		0 0 0 0 0 0 0 0 -1 1 0 2 3
χ_{112}		0 0 0 0 0 0 0 0 -1 -1 0 -2 -3
χ_{121}		0 0 0 0 0 0 0 0 1 -1 0 -4 -9

P_{BSQ}

P₀₀₀₀₀
P₁₀₀
P₁₀₁
P₁₀₋₁
P₁₀₂
P₀₁₁₁₀
P₀₁₁₁₁₁₁
P₁₁₁₁₀
P₁₁₁₁₁
P₁₁₁₁₁₋₁
P₁₁₂₁₀
P₁₁₂₁₁₁₁
P₁₁₃₁₁₁₁

Partial Pressures of $|B|=1, |S|=0$

Using the Boltzmann Approx

Through linear combinations of susceptibilities
(partial derivatives of the partition function)

we can build partial pressures.

$$P_{BSQ}$$

$$\underline{P_{100}} = \chi_2^B - \frac{1}{12}\chi_2^S + \frac{1}{12}\chi_4^S - \frac{1}{2}\chi_{31}^{BQ} + \frac{1}{2}\chi_{13}^{BQ} - \frac{1}{2}\chi_{13}^{BS} - \chi_{31}^{SQ} - \chi_{22}^{BQ} - \frac{3}{2}\chi_{22}^{BS} + \chi_{22}^{SQ} - \frac{1}{2}\chi_{211}^{BSQ} - \frac{3}{2}\chi_{121}^{BSQ}$$

$$\underline{P_{101}} = \frac{1}{6}\chi_2^S - \frac{1}{6}\chi_4^S + \chi_{31}^{BQ} - \frac{1}{2}\chi_{13}^{BQ} - \frac{1}{2}\chi_{13}^{BS} + \frac{1}{2}\chi_{31}^{SQ} + \frac{1}{2}\chi_{22}^{BQ} - \frac{1}{2}\chi_{22}^{BS} - \frac{1}{2}\chi_{22}^{SQ} + \chi_{211}^{BSQ} + \chi_{121}^{BSQ}$$

$$\underline{P_{1|1|0}} = \chi_{13}^{BS} + \chi_{31}^{SQ} + 2\chi_{22}^{BS} - \chi_{22}^{SQ} + \chi_{121}^{BSQ}$$

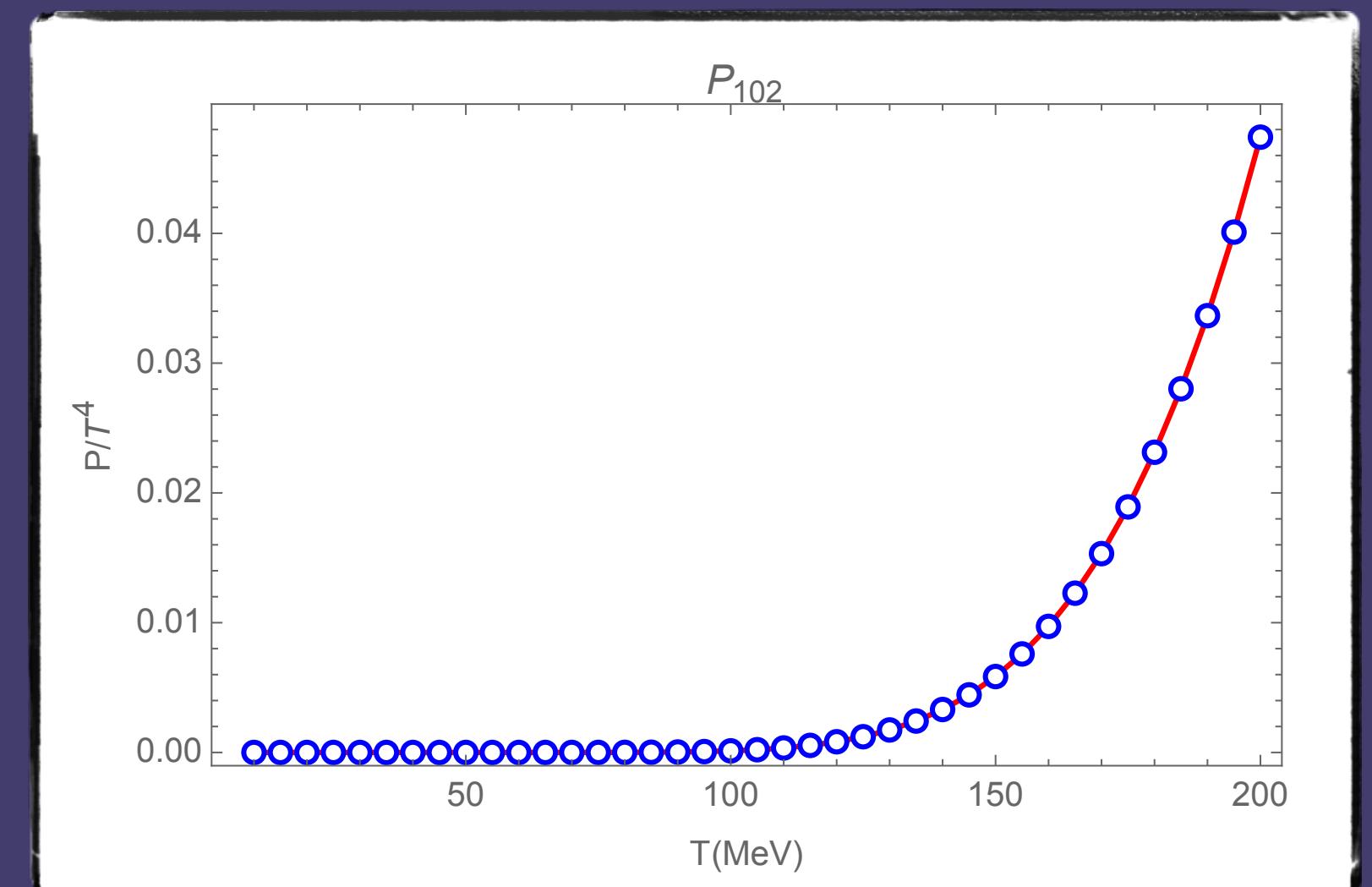
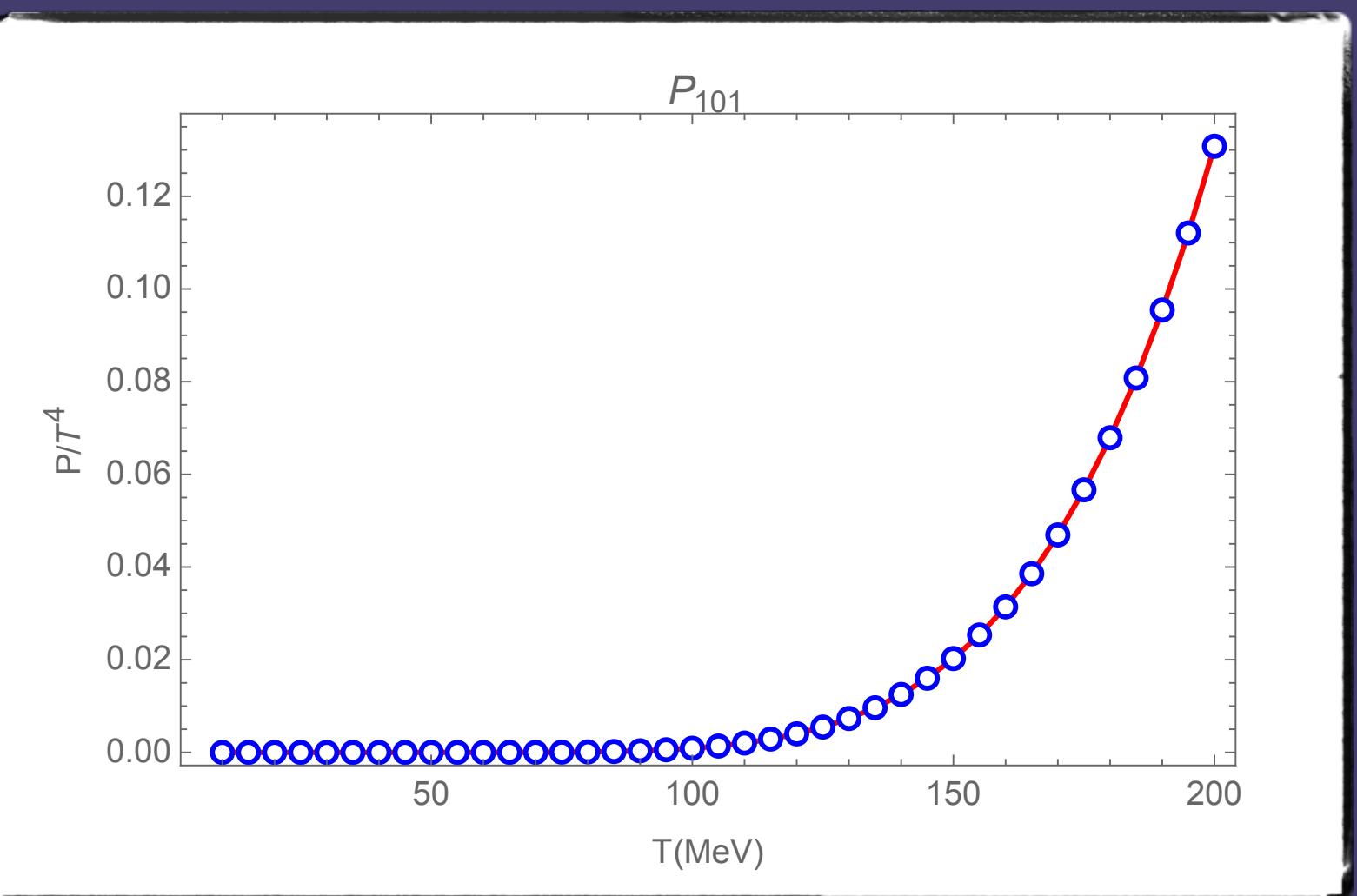
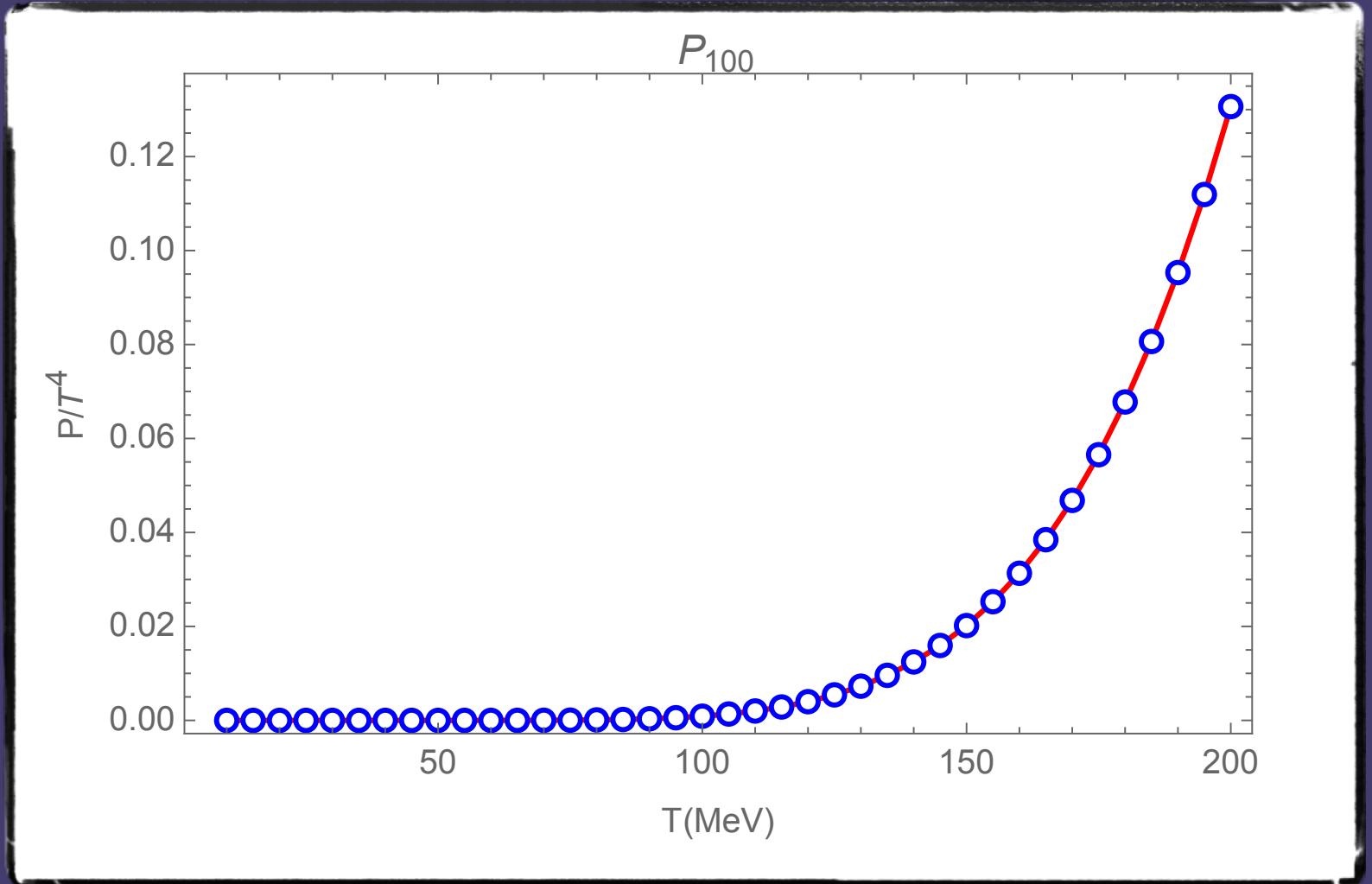
$$\underline{P_{102}} = -\frac{1}{6}\chi_{31}^{BQ} + \frac{1}{6}\chi_{13}^{BQ}$$

$$\chi_{mnk}^{BSQ} = \left. \frac{\partial^{(m+n+k)} [p(\hat{\mu}_B, \hat{\mu}_S, \hat{\mu}_Q)/T^4]}{\partial \hat{\mu}_B^m \partial \hat{\mu}_S^n \partial \hat{\mu}_Q^k} \right|_{\vec{\mu}=0}$$

$$\ln Z^{HRG}(T, V, \boldsymbol{\mu}) = \sum_{i \in PDG} \ln Z_i(T, V, \boldsymbol{\mu})$$

HRG Partial Pressure Plots

$$\ln Z^{HRG}(T, V, \mu) = \sum_{i \in PDG} \ln Z_i(T, V, \mu)$$



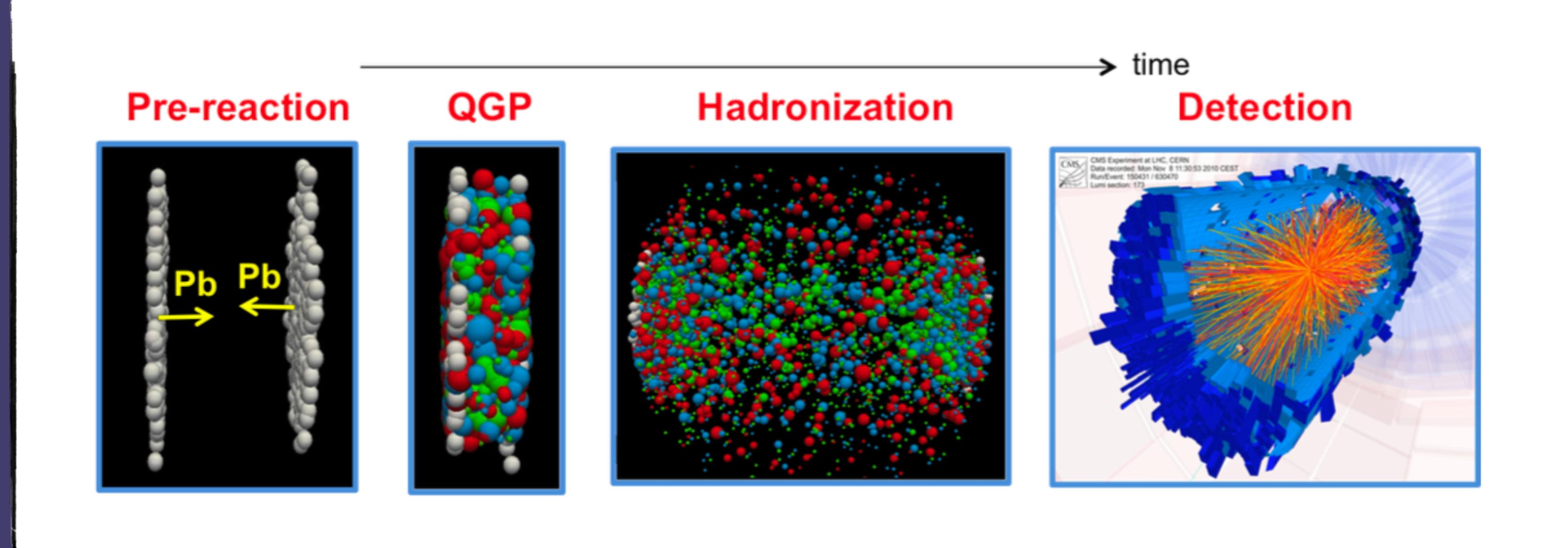
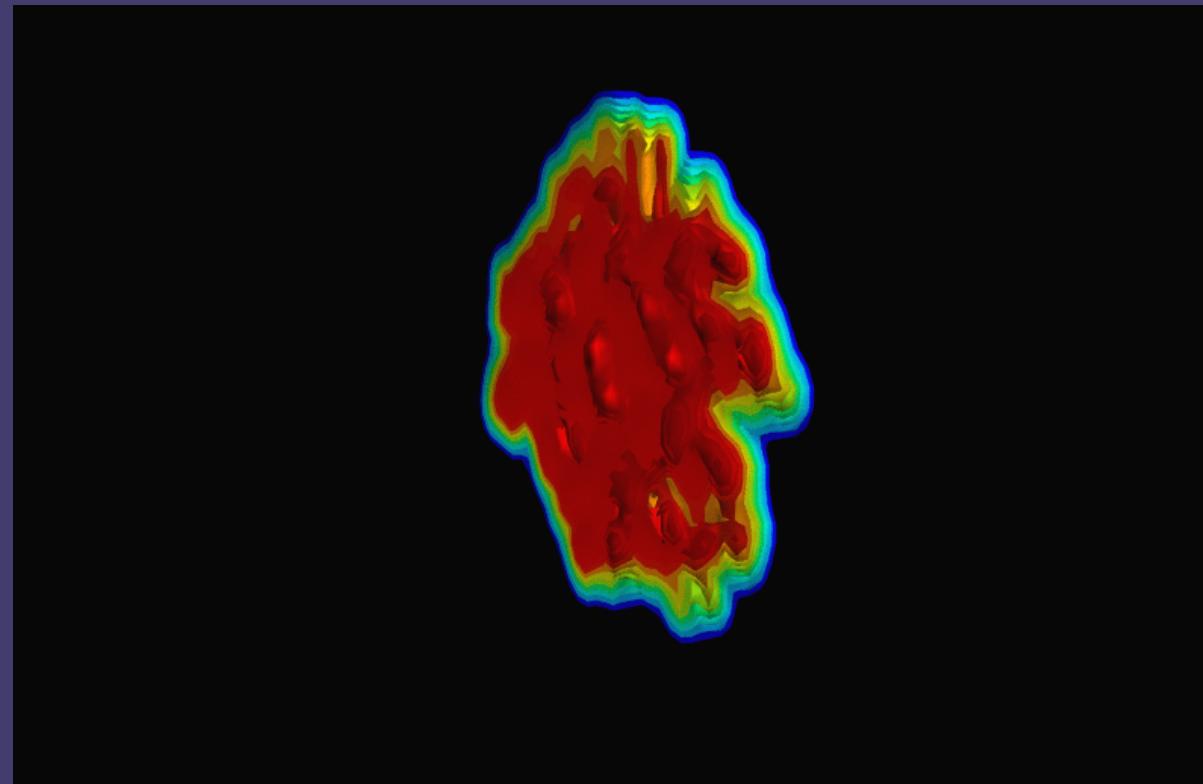
- **Linear Combo**
- $P_{Partial} = \sum_{i \in PDG} \sum_s c_s \chi_s^{HRG}$
- **Sublist**
- $P_{Partial} = \sum_{i \in Sublist} \ln Z_i(T, V, \mu)$

Quark Gluon Plasma

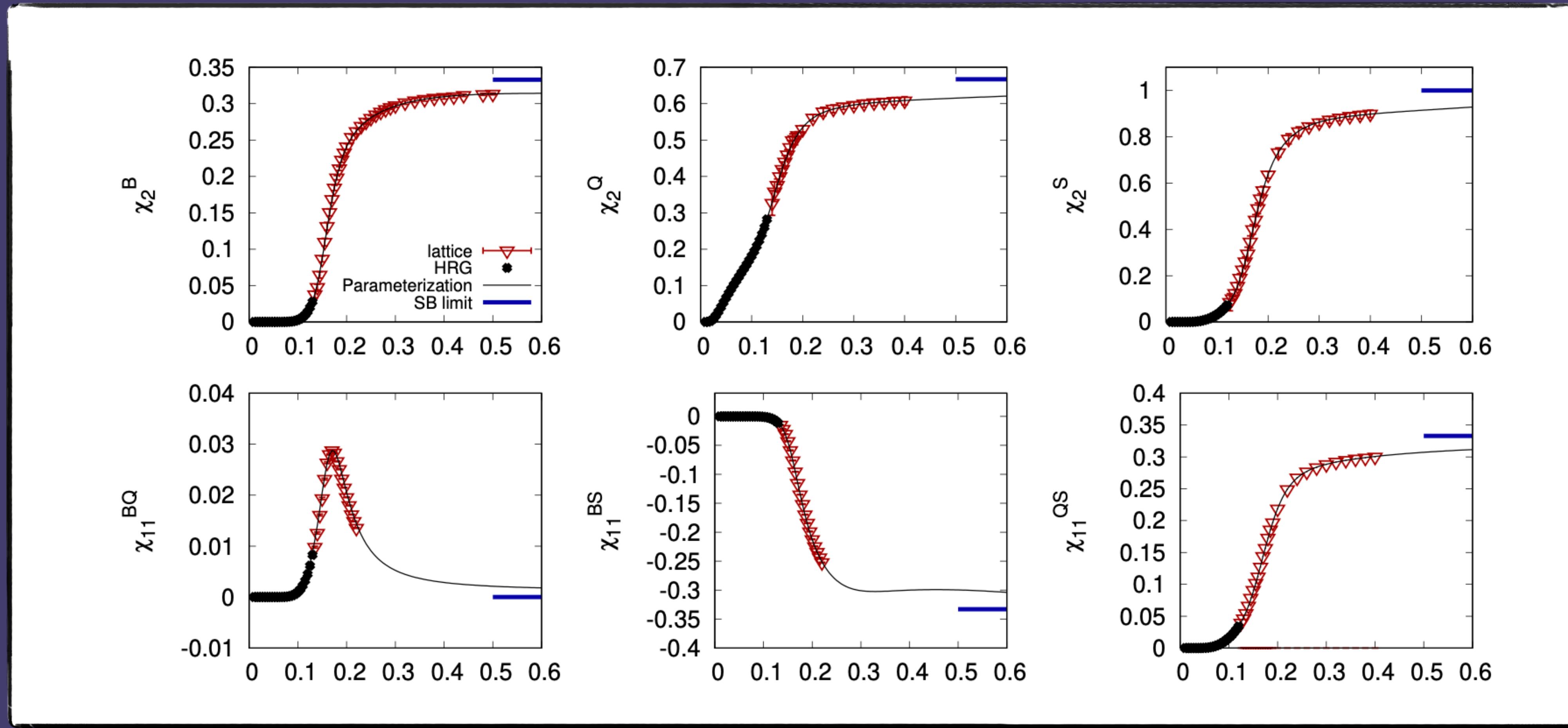
We know that beyond the deconfinement temperature quarks and gluons, which are typically bound in hadron states, transition into the exotic phase known as quark gluon plasma. Therefore, they will no longer contribute to that hadron family's partial pressure.

In other words the Stefan-Boltzmann limit for each partial pressures should be zero.

Collision Simulation



Steffan-Boltzmann limits



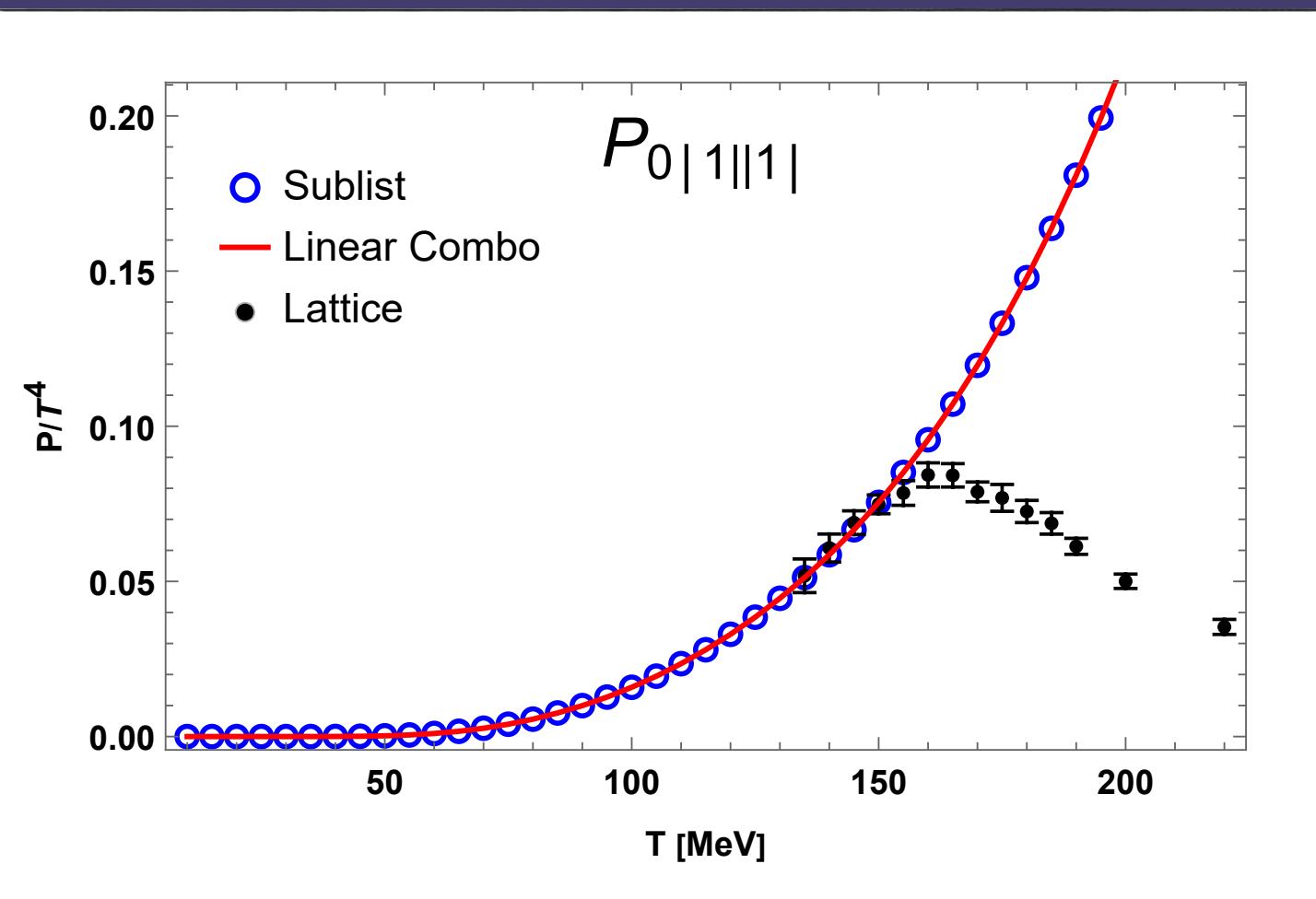
Susceptibilities in HRG

$$\begin{aligned}
 P(\hat{\mu}_B, \hat{\mu}_S, \hat{\mu}_Q) = & P_{000} + P_{00|1|} \cosh(\hat{\mu}_Q) + P_{100} \cosh(\hat{\mu}_B) \\
 & + P_{101} \cosh(\hat{\mu}_B + \hat{\mu}_Q) + P_{10-1} \cosh(\hat{\mu}_B - \hat{\mu}_Q) \\
 & + P_{102} \cosh(\hat{\mu}_B + 2\hat{\mu}_Q) + P_{0|1|0} \cosh(\hat{\mu}_S) \\
 & + P_{0|1||1|} \cosh(\hat{\mu}_S + \hat{\mu}_Q) + P_{1|1|0} \cosh(\hat{\mu}_B - \hat{\mu}_S) \\
 & + P_{1|1|1} \cosh(\hat{\mu}_B - \hat{\mu}_S + \hat{\mu}_Q) \\
 & + P_{1|1|-1} \cosh(\hat{\mu}_B - \hat{\mu}_S - \hat{\mu}_Q) \\
 & + P_{1|2|0} \cosh(\hat{\mu}_B - 2\hat{\mu}_S) \\
 & + P_{1|2||1|} \cosh(\hat{\mu}_B - 2\hat{\mu}_S - \hat{\mu}_Q) \\
 & + P_{1|3||1|} \cosh(\hat{\mu}_B - 3\hat{\mu}_S - \hat{\mu}_Q).
 \end{aligned}$$

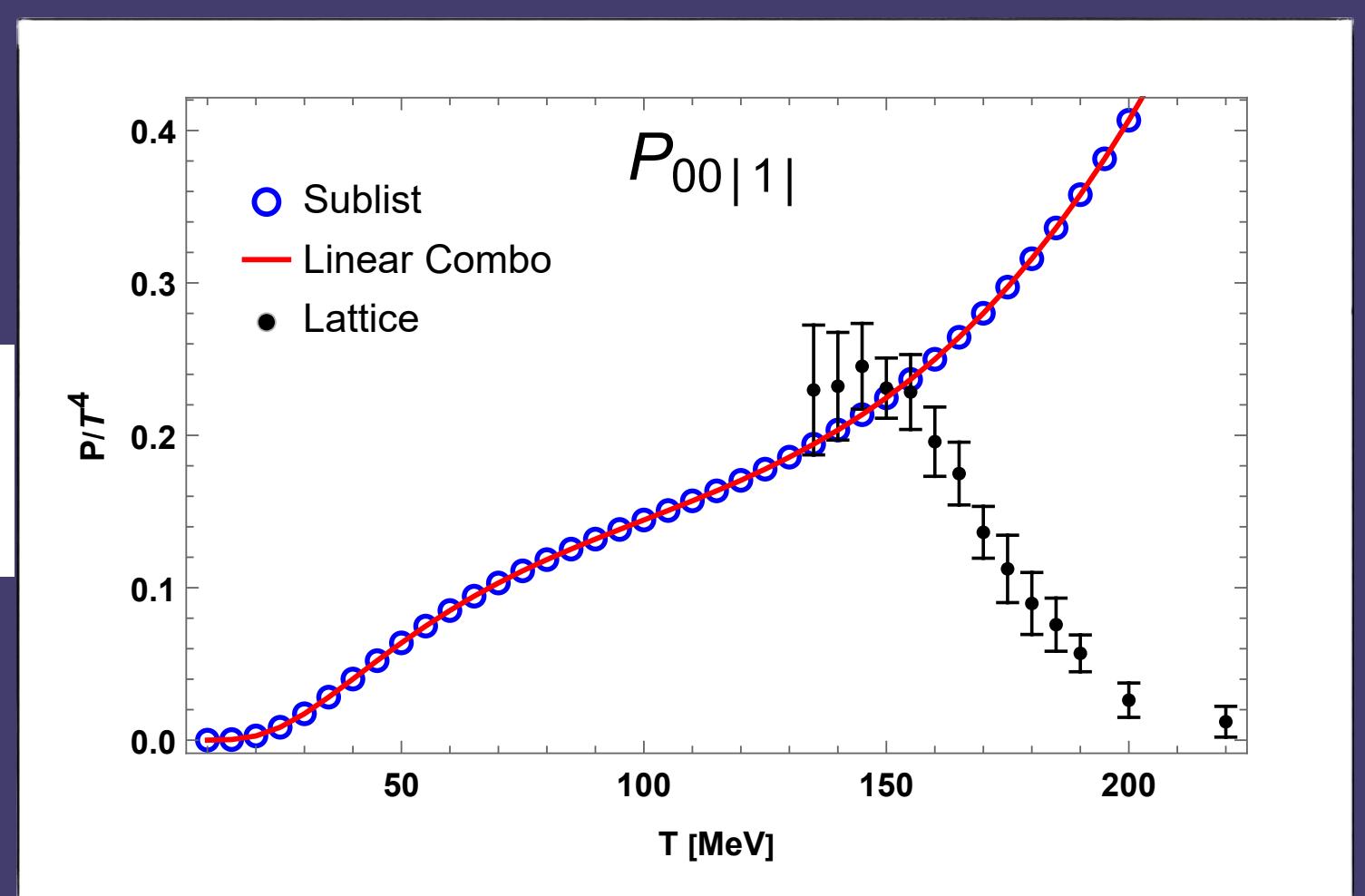
χ_{BSQ}	χ_{BSQ}	P_{BSQ}
χ_{100}	1 χ_{100}^{sb}	$P_{SB} = 0$
χ_{002}	2 χ_{002}^{sb}	P_{00III}
χ_{020}	3 χ_{020}^{sb}	P_{100}
χ_{101}	4 χ_{101}^{sb}	P_{101}
χ_{110}	5 χ_{110}^{sb}	P_{10-1}
χ_{011}	6 χ_{011}^{sb}	P_{102}
χ_{400}	7 χ_{400}^{sb}	P_{0110}
χ_{004}	8 χ_{004}^{sb}	P_{0111}
χ_{040}	9 χ_{040}^{sb}	P_{1110}
χ_{301}	10 χ_{301}^{sb}	P_{11110}
χ_{310}	11 χ_{310}^{sb}	P_{011111}
χ_{013}	12 χ_{013}^{sb}	P_{11110}
χ_{103}	13 χ_{103}^{sb}	P_{11111}
χ_{130}	14 χ_{130}^{sb}	$P_{11111-1}$
χ_{031}	15 χ_{031}^{sb}	P_{11210}
χ_{202}	16 χ_{202}^{sb}	P_{112100}
χ_{220}	17 χ_{220}^{sb}	P_{113100}
χ_{022}	18 χ_{022}^{sb}	14
χ_{211}	19 χ_{211}^{sb}	
χ_{121}	20 χ_{121}^{sb}	
χ_{112}	21 χ_{112}^{sb}	

Mesons

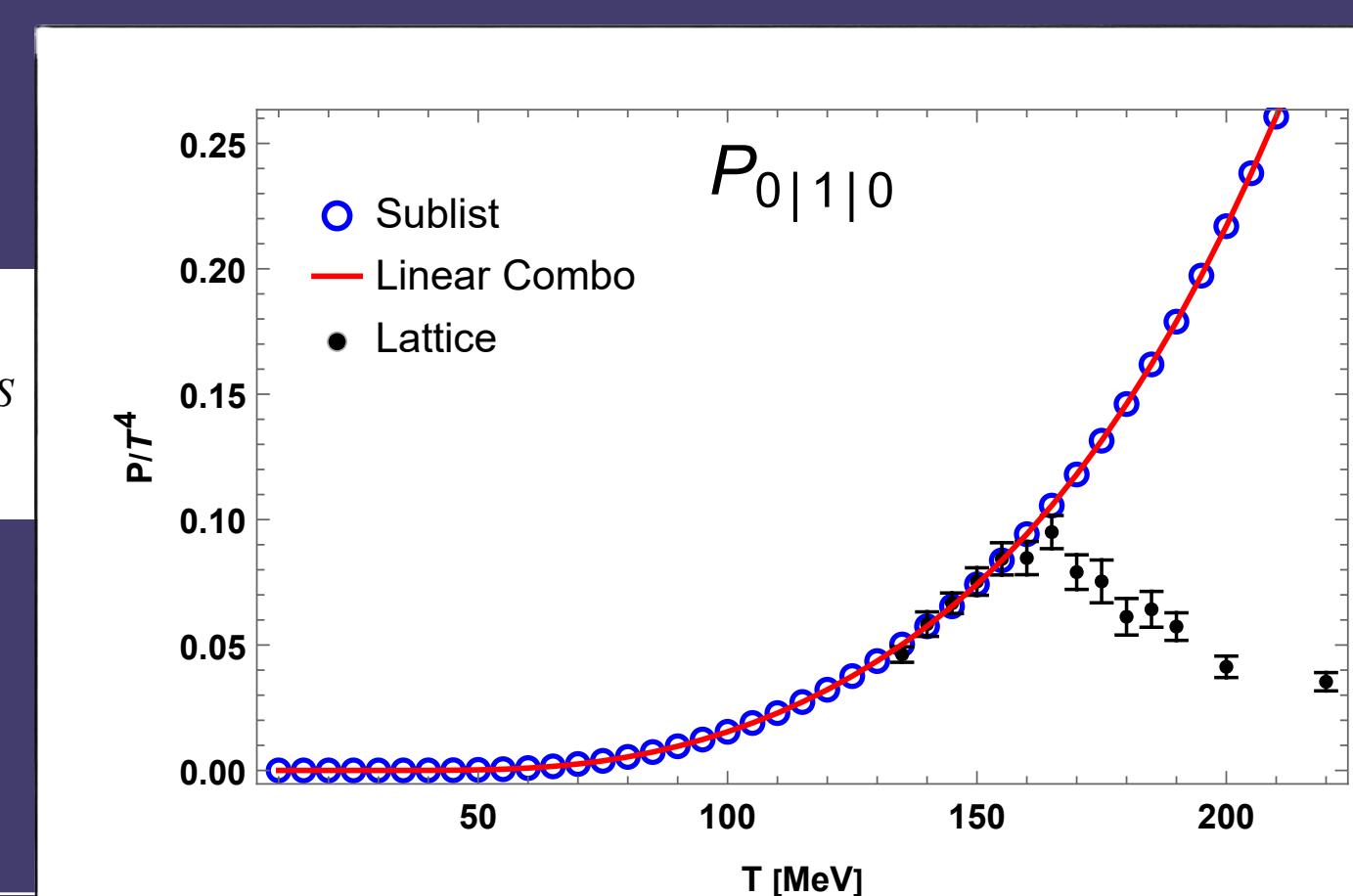
$$P_{00|1|} = \chi_2^Q + \frac{1}{8}\chi_2^S - \frac{3}{2}\chi_{22}^{SQ} + \frac{1}{2}\chi_{31}^{SQ} - \frac{1}{8}\chi_4^S - \frac{3}{2}\chi_{112}^{BSQ} - \frac{1}{4}\chi_{13}^{BS} - \frac{19}{8}\chi_2^B - \chi_{22}^{BQ} - \frac{1}{2}\chi_{211}^{BSQ} + \frac{1}{4}\chi_{31}^{BS} + \frac{19}{8}\chi_4^B$$



$$P_{0|1||1|} = \frac{1}{8}\chi_2^Q - \frac{1}{8}\chi_4^Q + \frac{1}{3}\chi_{13}^{SQ} - \frac{1}{12}\chi_2^S + \chi_{22}^{SQ} - \frac{1}{3}\chi_{31}^{SQ} + \frac{1}{12}\chi_4^S + \frac{1}{4}\chi_{13}^{BQ} + \chi_{112}^{BSQ} + \frac{1}{6}\chi_{13}^{BS} - \frac{1}{4}\chi_{31}^{BQ} - \frac{1}{6}\chi_{31}^{BS}$$



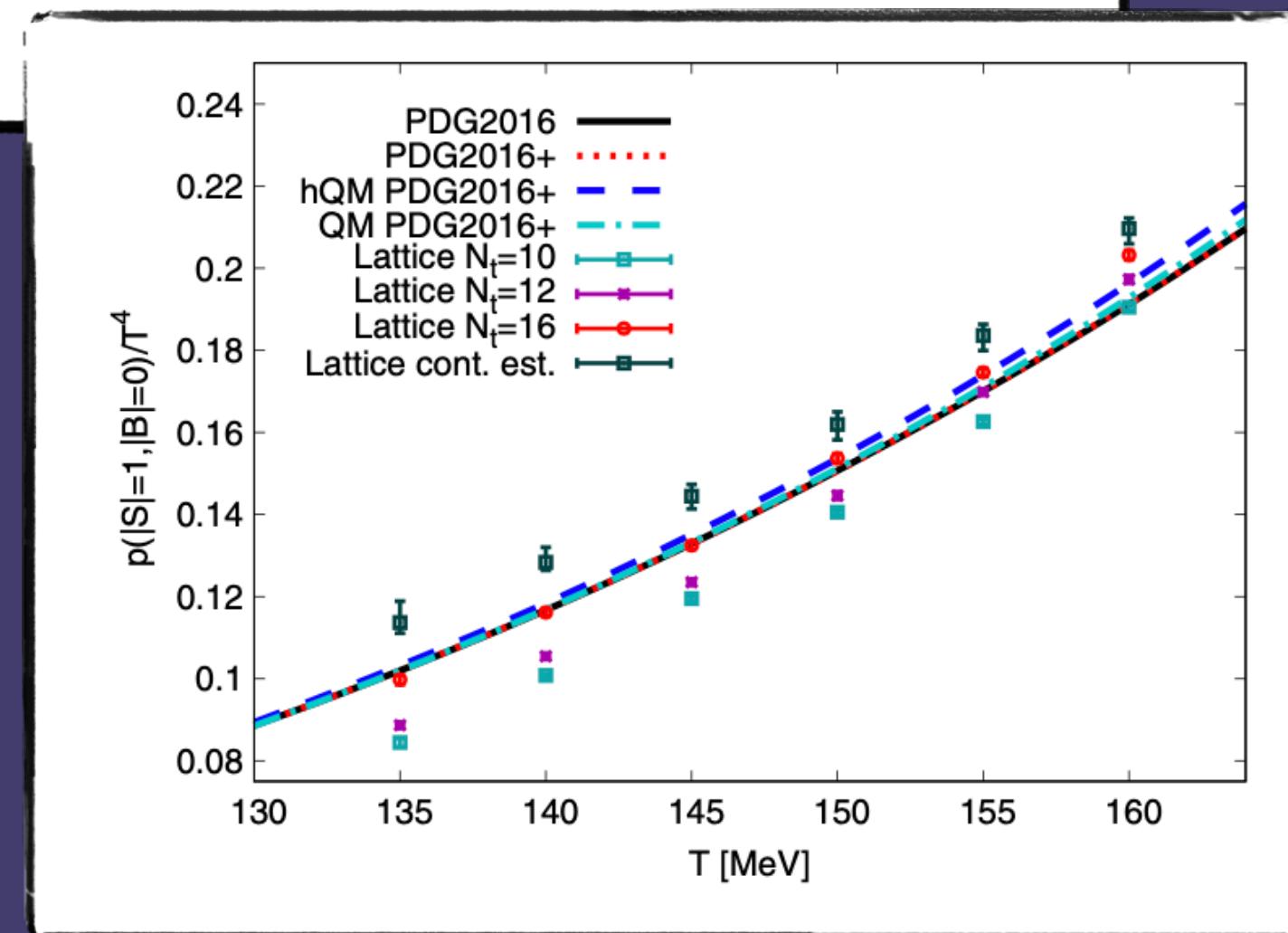
$$P_{0|1|0} = -\frac{1}{3}\chi_{13}^{SQ} + \chi_2^S - \chi_{22}^{SQ} + \frac{1}{3}\chi_{31}^{SQ} + 3\chi_{11}^{BS} - \chi_{112}^{BSQ} + \frac{1}{3}\chi_{13}^{BS} - \frac{7}{3}\chi_{31}^{BS}$$



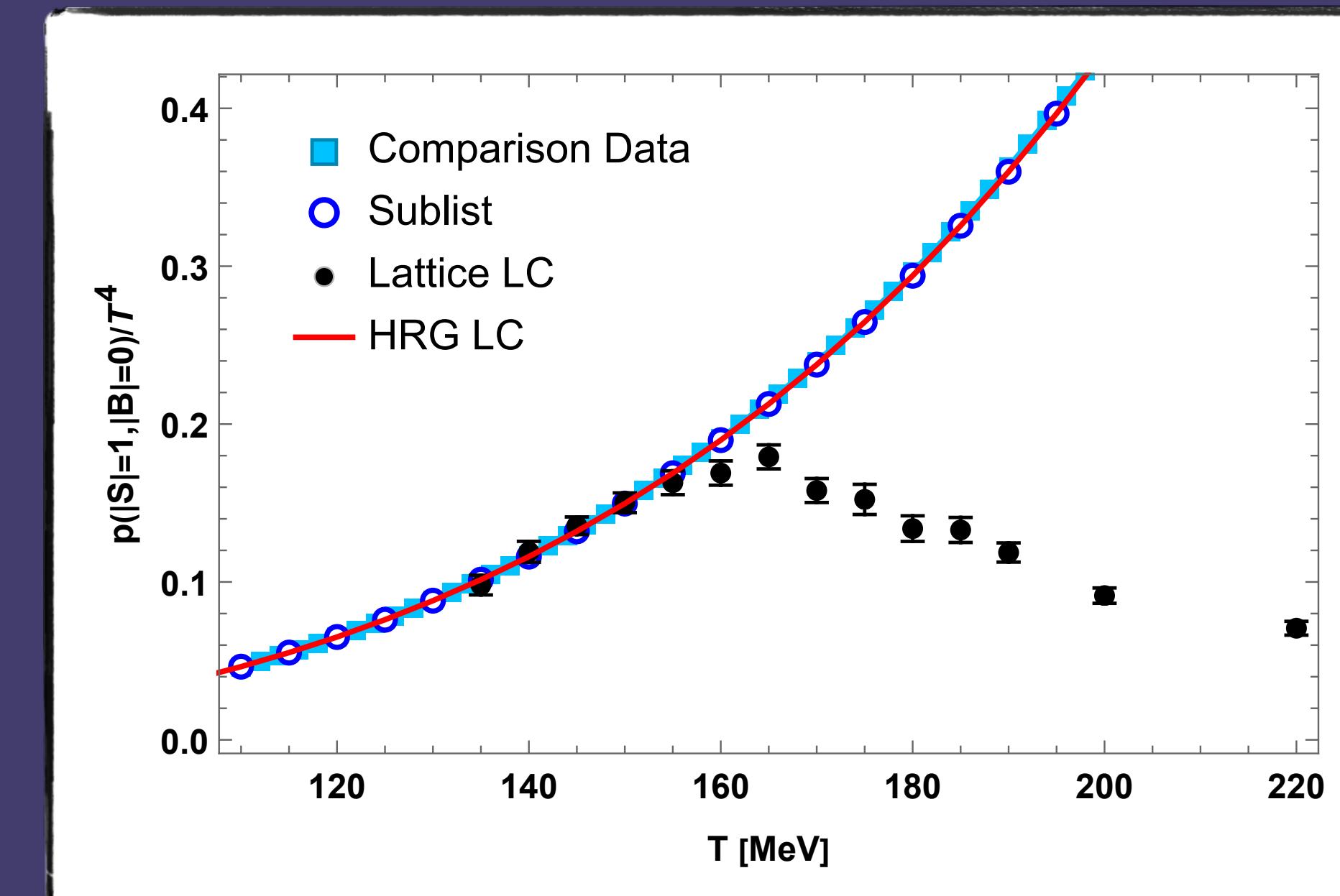
Meson Comparison Plots

Full Pressure (2D)

$$\begin{aligned}
 P(\hat{\mu}_B, \hat{\mu}_S) = & P_{00}^{BS} + P_{10}^{BS} \cosh(\hat{\mu}_B) + P_{01}^{BS} \cosh(-\hat{\mu}_S) \\
 & + P_{11}^{BS} \cosh(\hat{\mu}_B - \hat{\mu}_S) + \cancel{P_{12}^{BS} \cosh(\hat{\mu}_B - 2\hat{\mu}_S)} \\
 & + P_{13}^{BS} \cosh(\hat{\mu}_B - 3\hat{\mu}_S),
 \end{aligned}$$

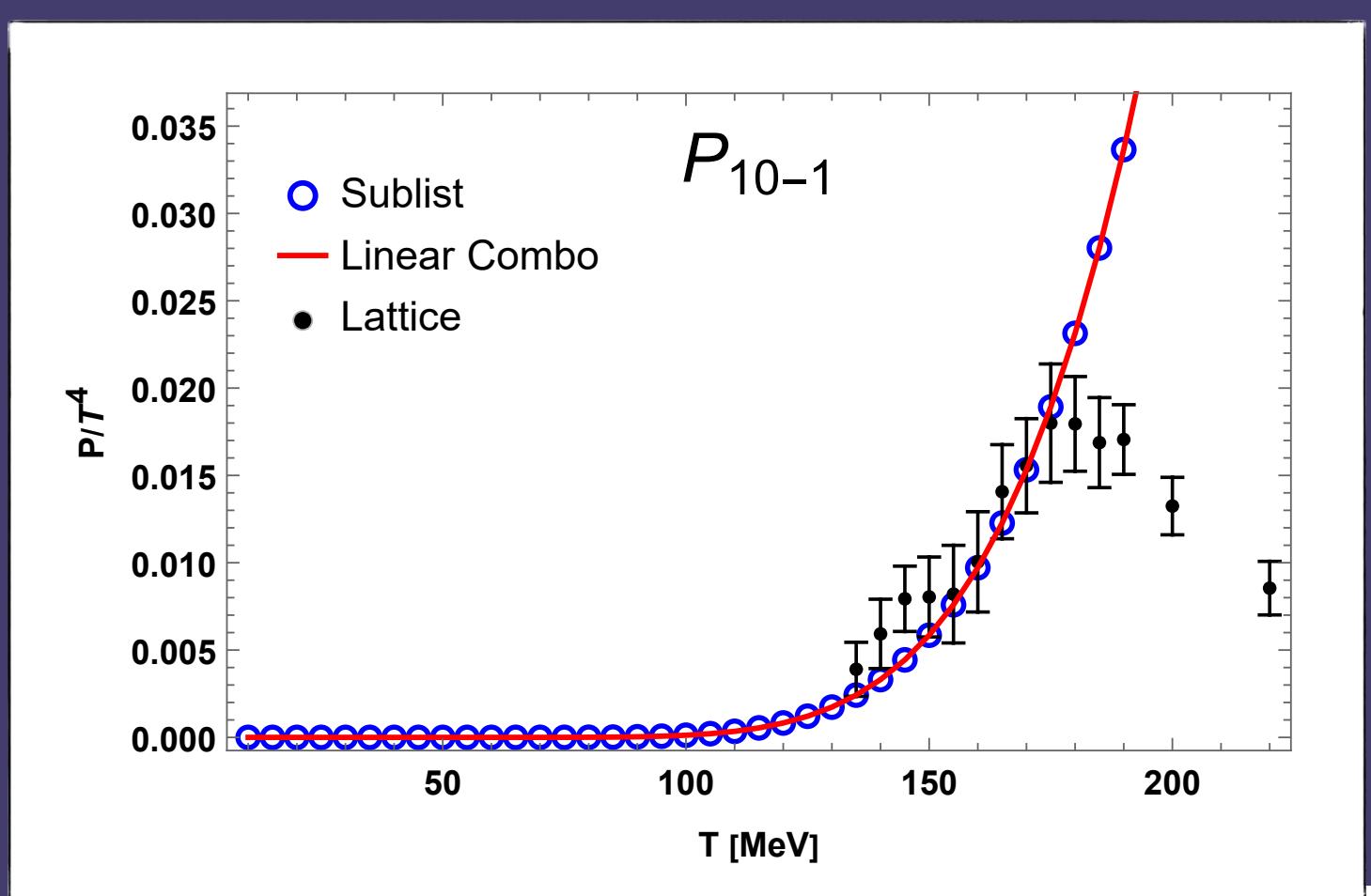


Comparing our Linear Combinations (summed over Q) to 2D version
with PDG2016+ data



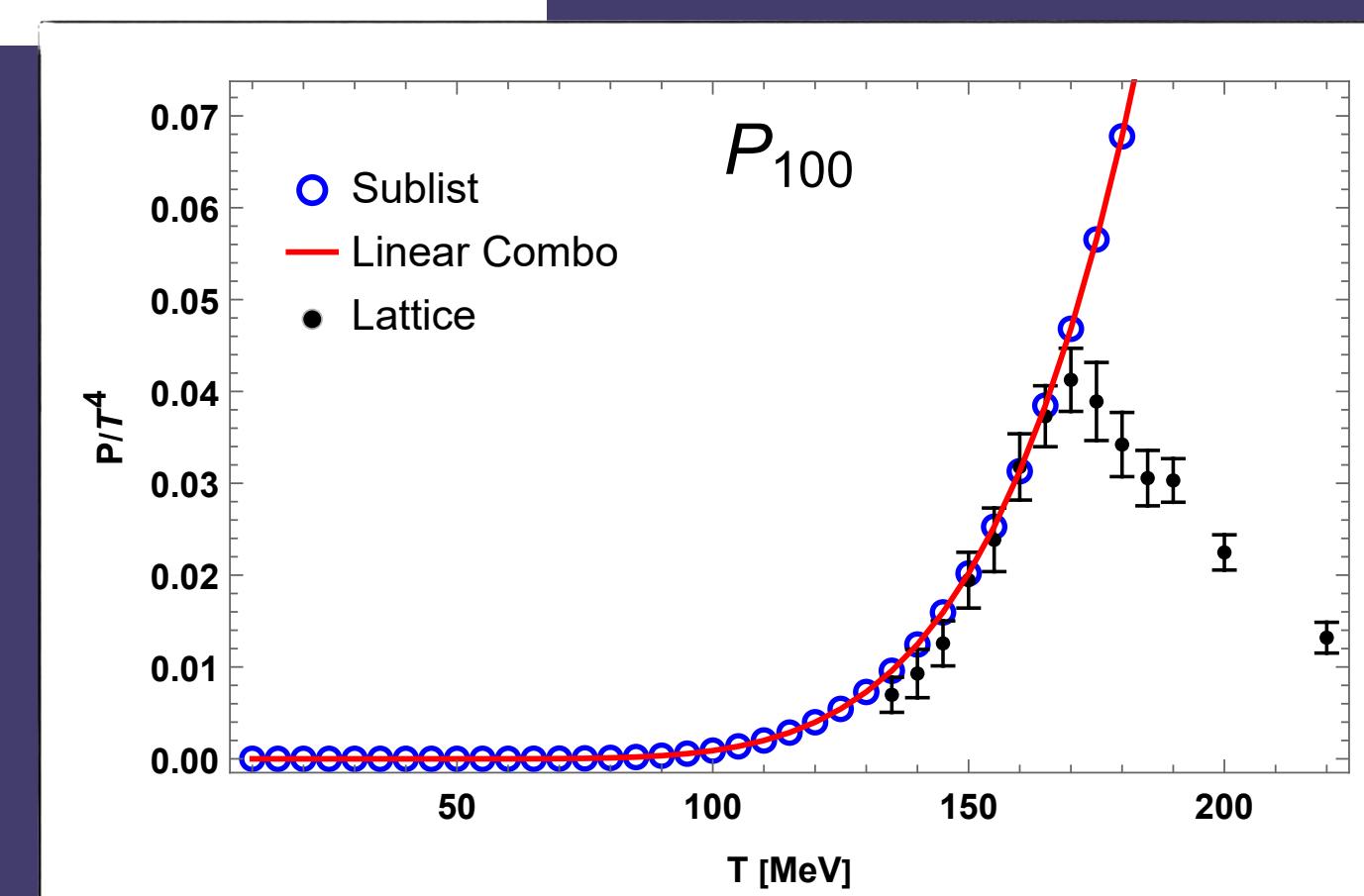
Partial pressures for various particle lists compared to Lattice

$B = 0$
 $|S| = 1$
 $|Q| = 0, 1$



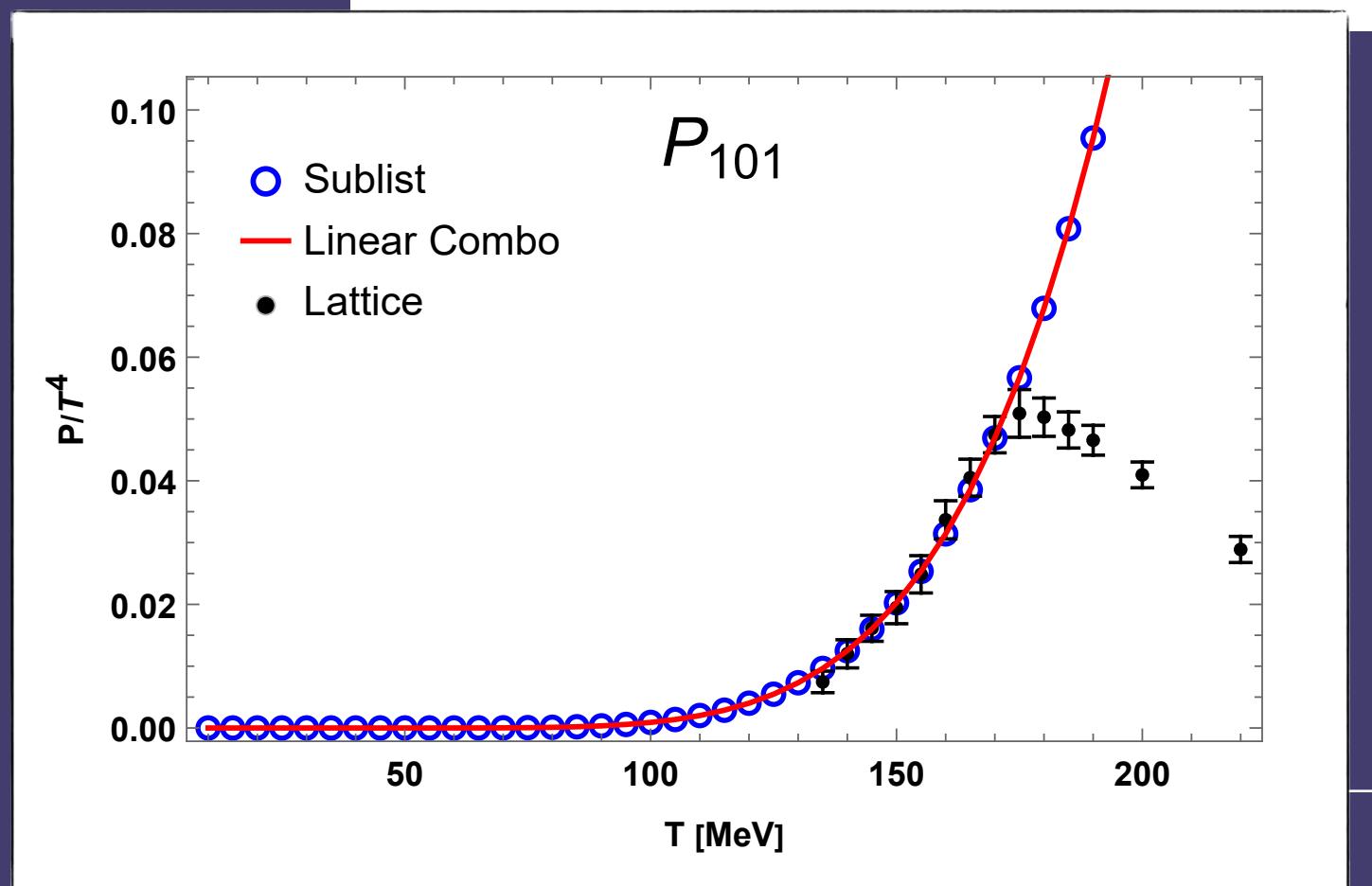
$$P_{100} = -\frac{1}{2}\chi_{11}^{BQ} + \frac{1}{2}\chi_{13}^{BQ} + \chi_{11}^{BS} - \chi_{112}^{BSQ} + \frac{1}{2}\chi_{121}^{BSQ} + \chi_2^B - \chi_{22}^{BQ} + \frac{1}{2}\chi_{211}^{BSQ} + \frac{1}{2}\chi_{22}^{BS} + \frac{1}{2}\chi_{31}^{BS}$$

Baryons (S=0)

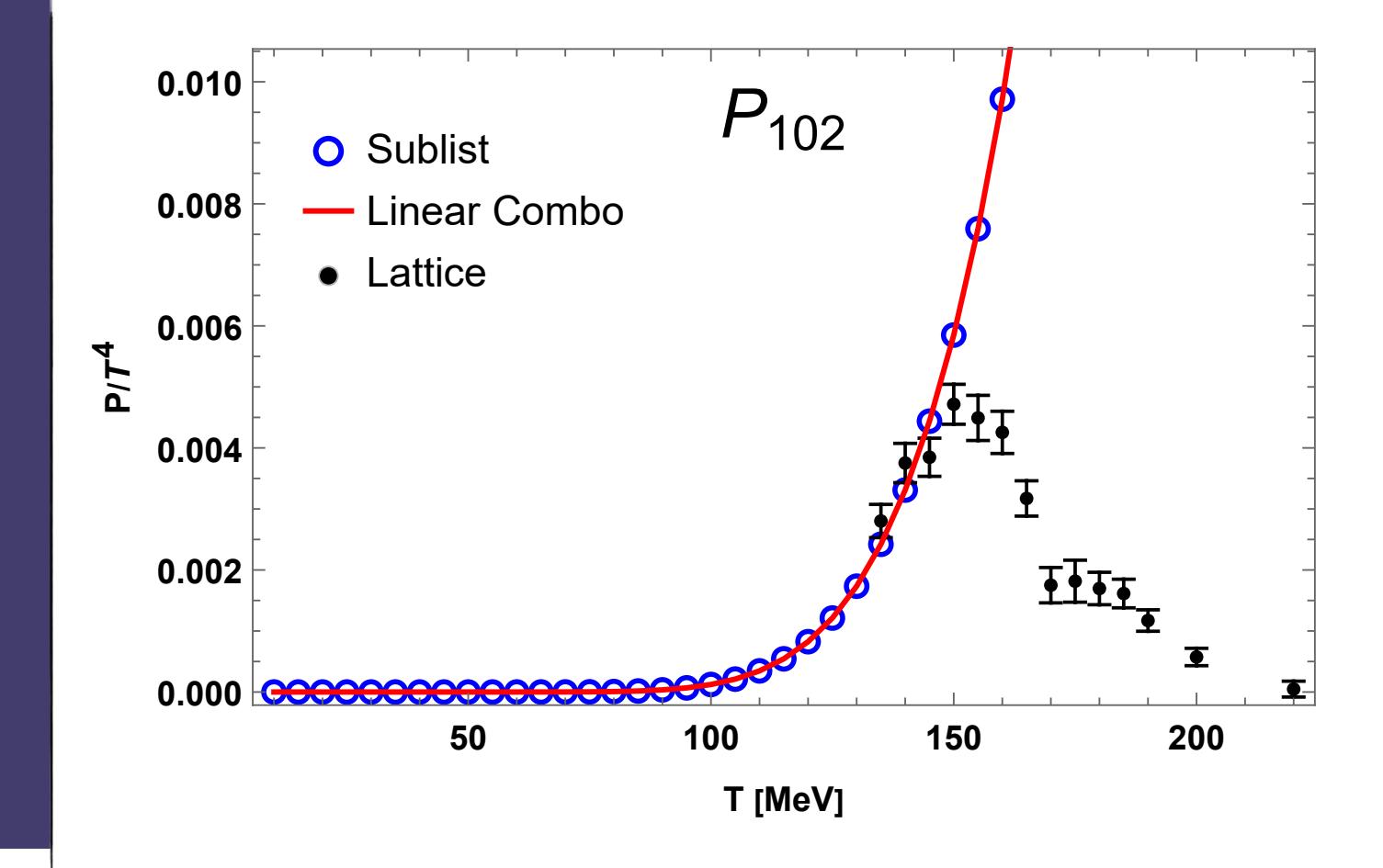


$$P_{10-1} = -\frac{1}{2}\chi_{13}^{SQ} + \frac{1}{2}\chi_{22}^{SQ} - \frac{1}{6}\chi_{13}^{BQ} + \frac{1}{3}\chi_{11}^{BS} + \chi_{112}^{BSQ} + \frac{1}{6}\chi_{13}^{BS} + \frac{4-3\pi^2}{6-9\pi^2}\chi_2^B + \frac{1}{2}\chi_{22}^{BQ} + \frac{1}{2}\chi_{22}^{BS} - \frac{1}{3}\chi_{31}^{BQ} + \frac{4-3\pi^2}{-6+9\pi^2}\chi_4^B$$

$$P_{101} = -\frac{1}{8}\chi_2^S + \frac{1}{2}\chi_{22}^{SQ} - \frac{1}{2}\chi_{31}^{SQ} + \frac{1}{8}\chi_4^S + \chi_{11}^{BQ} - \frac{1}{2}\chi_{13}^{BQ} + \chi_{112}^{BSQ} - \chi_{121}^{BSQ} + \frac{1}{4}\chi_{13}^{BS} + \frac{3}{8}\chi_2^B + \frac{1}{2}\chi_{22}^{BQ} - \frac{1}{4}\chi_{31}^{BS} - \frac{3}{8}\chi_4^B$$



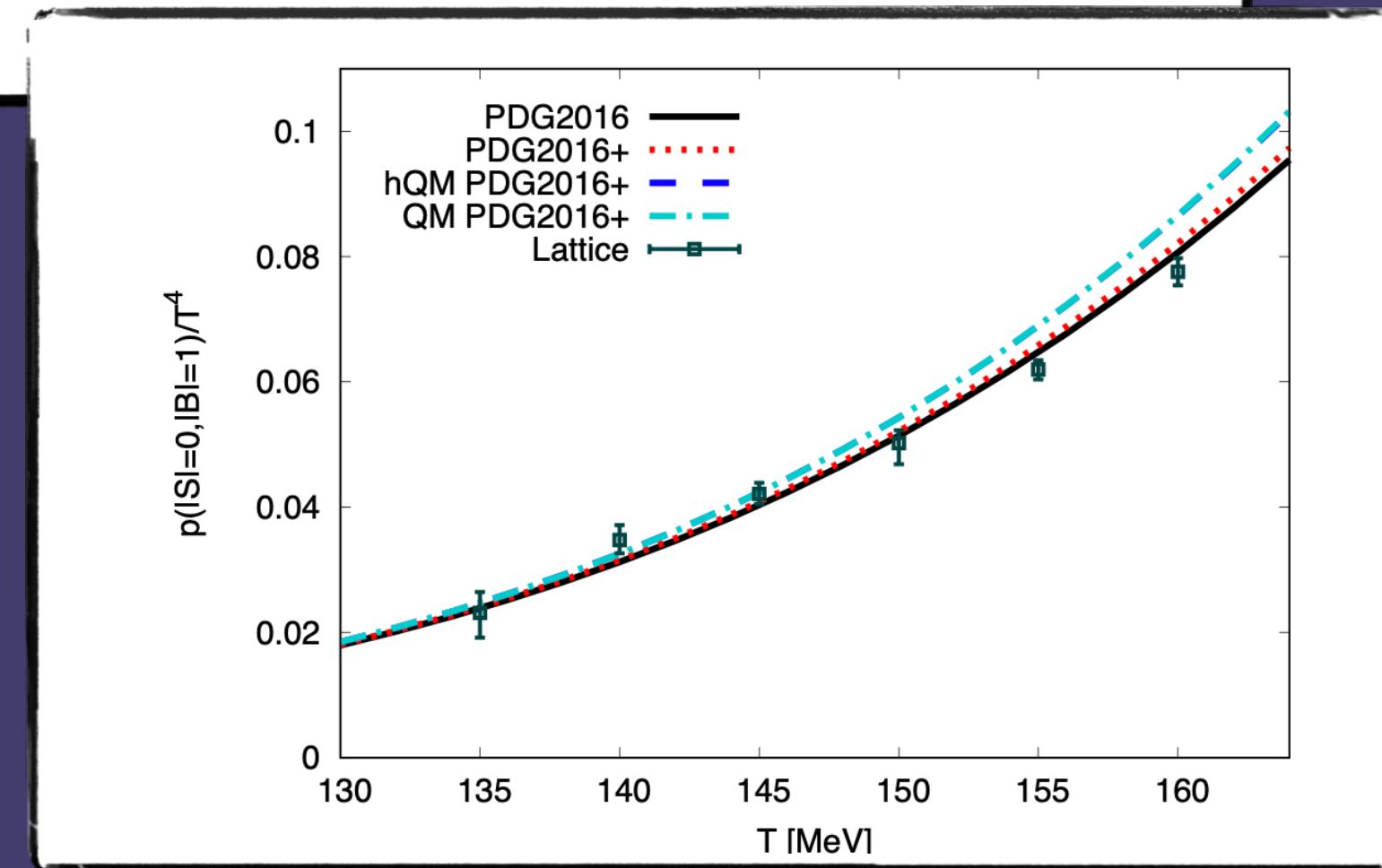
$$P_{102} = \frac{2}{-6+9\pi^2}\chi_{13}^{SQ} + \frac{1}{2-3\pi^2}\chi_{22}^{SQ} + \frac{1}{-6+9\pi^2}\chi_{31}^{SQ} + \frac{1}{6}\chi_{13}^{BQ} + \frac{2}{-6+9\pi^2}\chi_{11}^{BS} + \frac{1}{2-3\pi^2}\chi_{112}^{BSQ} + \frac{1}{-6+9\pi^2}\chi_{13}^{BS} + \frac{1}{2-3\pi^2}\chi_{211}^{BSQ} + \frac{1}{-2+3\pi^2}\chi_{22}^{BS} - \frac{1}{6}\chi_{31}^{BQ}$$



Baryons ($S=0$) Comparison Plots

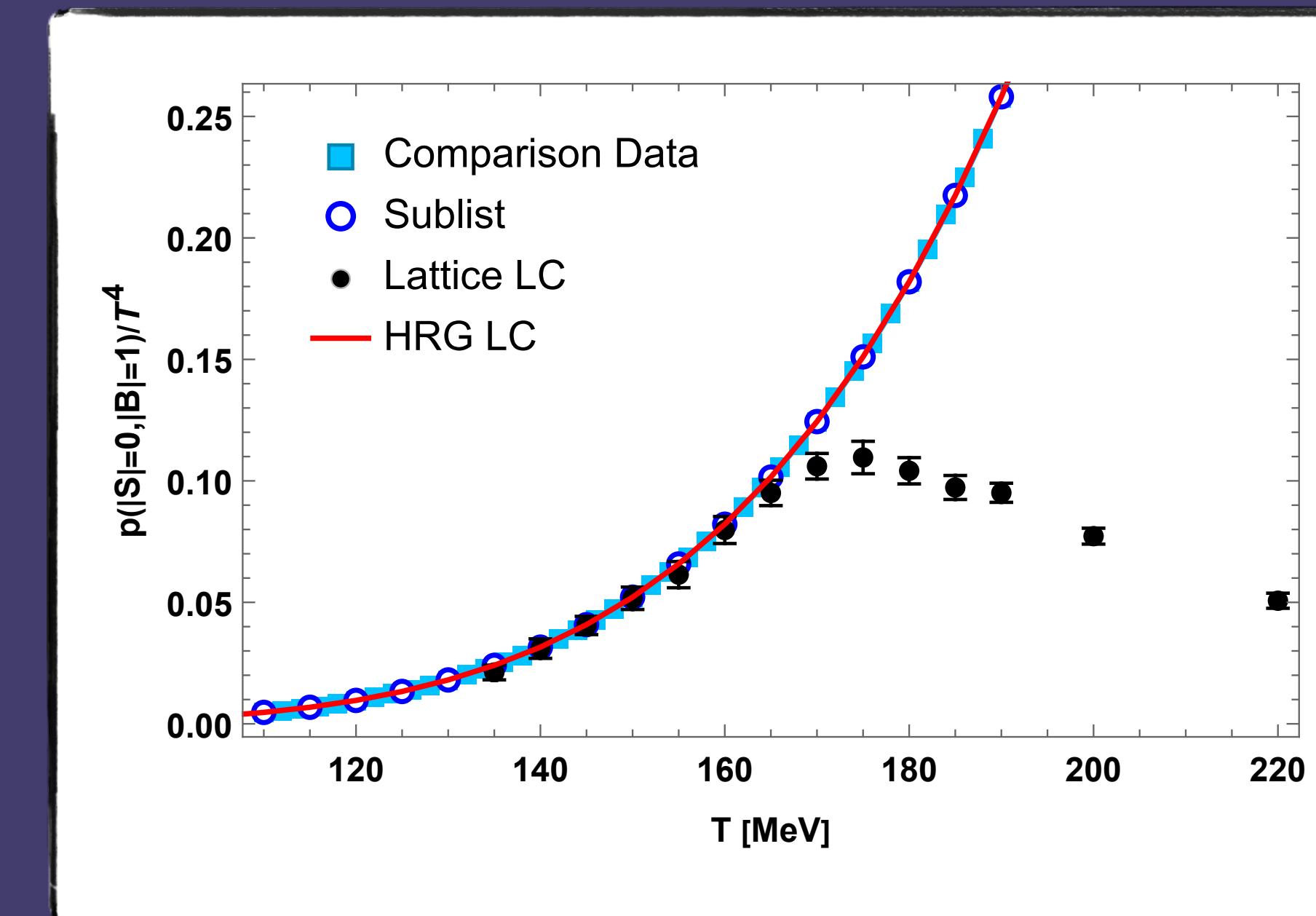
Full Pressure (2D)

$$\begin{aligned}
 P(\hat{\mu}_B, \hat{\mu}_S) = & P_{00}^{BS} + P_{10}^{BS} \cosh(\hat{\mu}_B) + P_{01}^{BS} \cosh(-\hat{\mu}_S) \\
 & + P_{11}^{BS} \cosh(\hat{\mu}_B - \hat{\mu}_S) + P_{12}^{BS} \cosh(\hat{\mu}_B - 2\hat{\mu}_S) \\
 & + P_{13}^{BS} \cosh(\hat{\mu}_B - 3\hat{\mu}_S),
 \end{aligned}$$



Partial pressures for various particle lists compared to Lattice

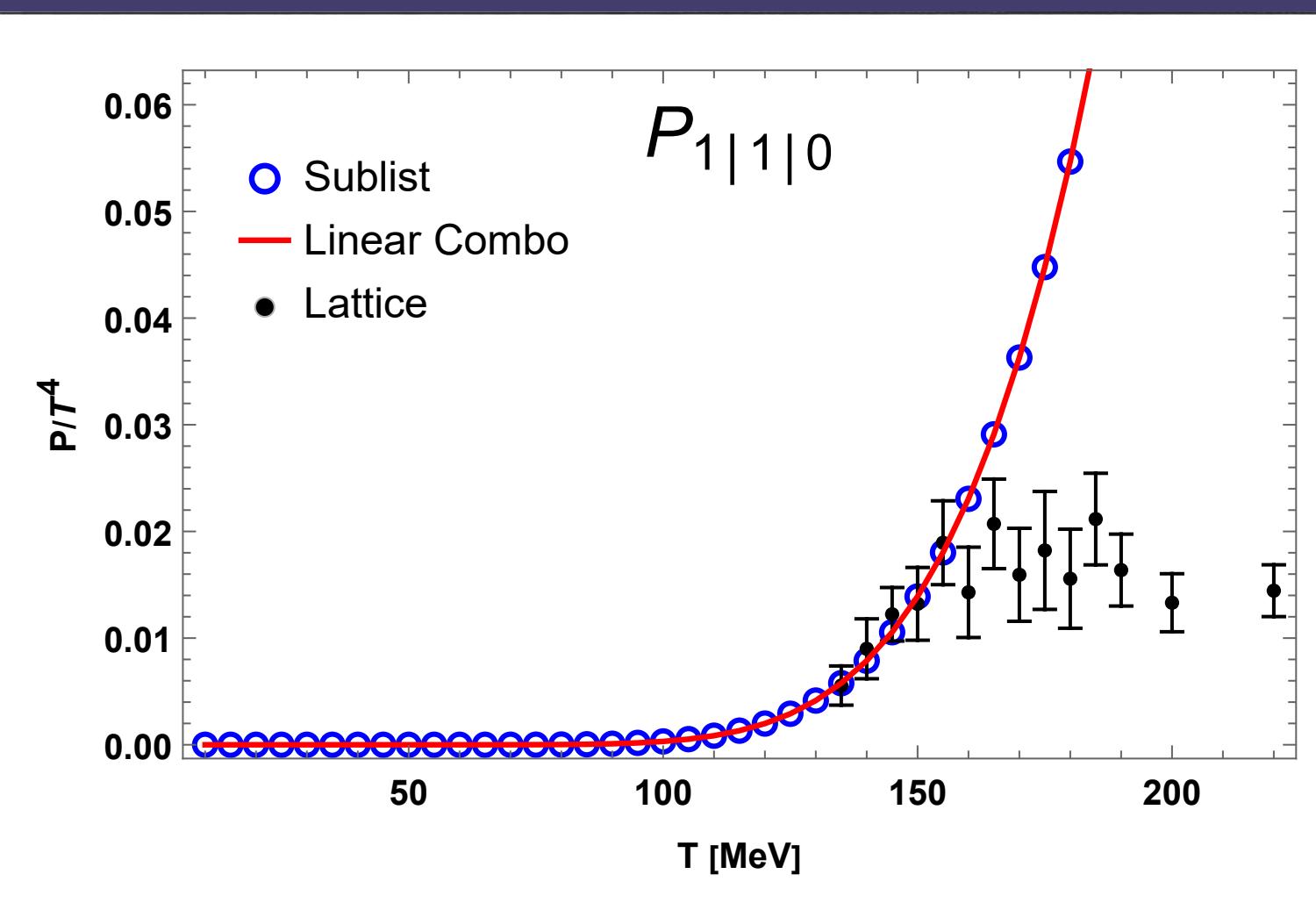
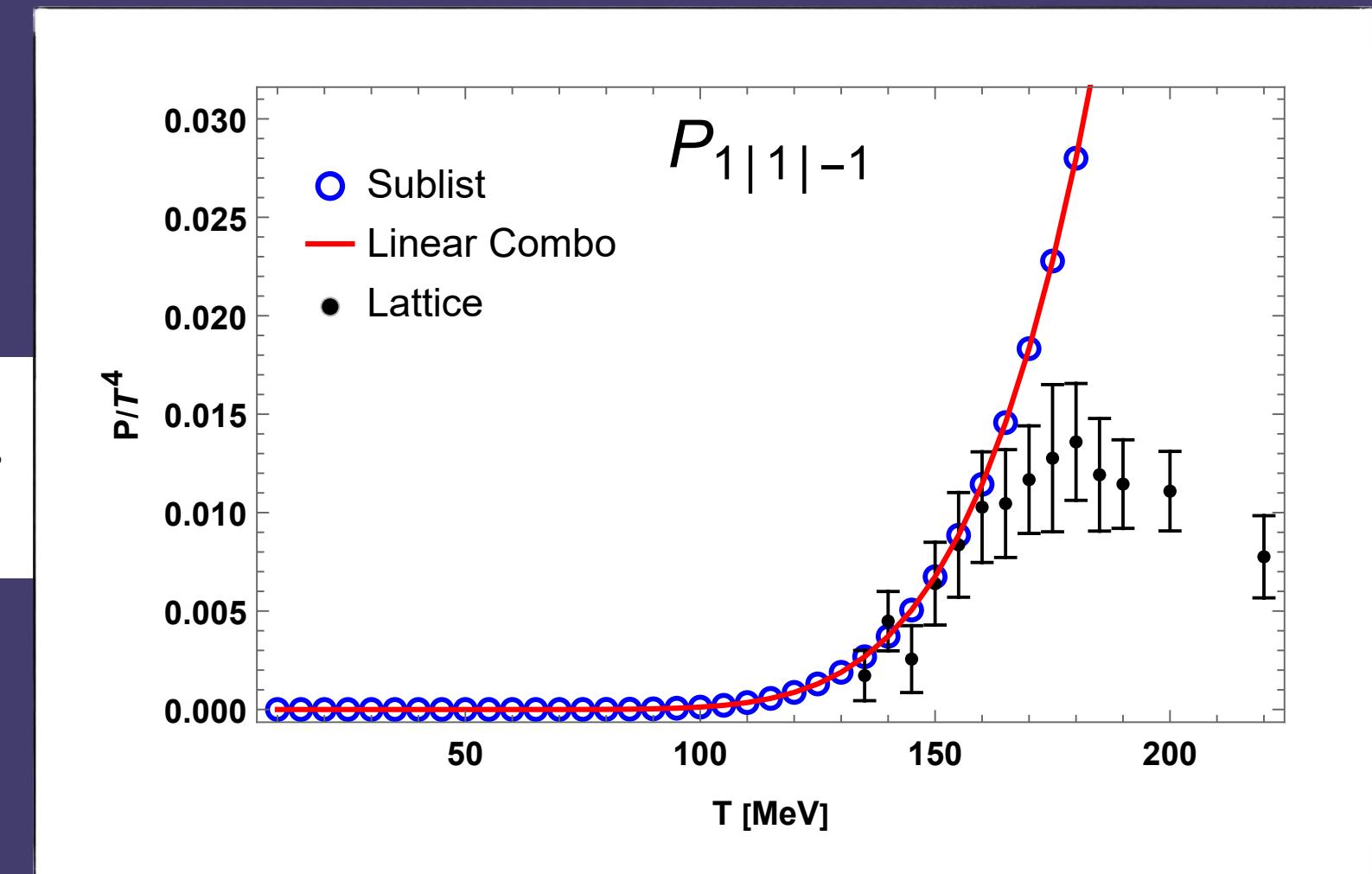
Comparing our Linear Combinations (summed over Q) to 2D version
with PDG2016+ data



$|B| = 1$
 $S = 0$
 $|Q| = -1, 0, 1, 2$

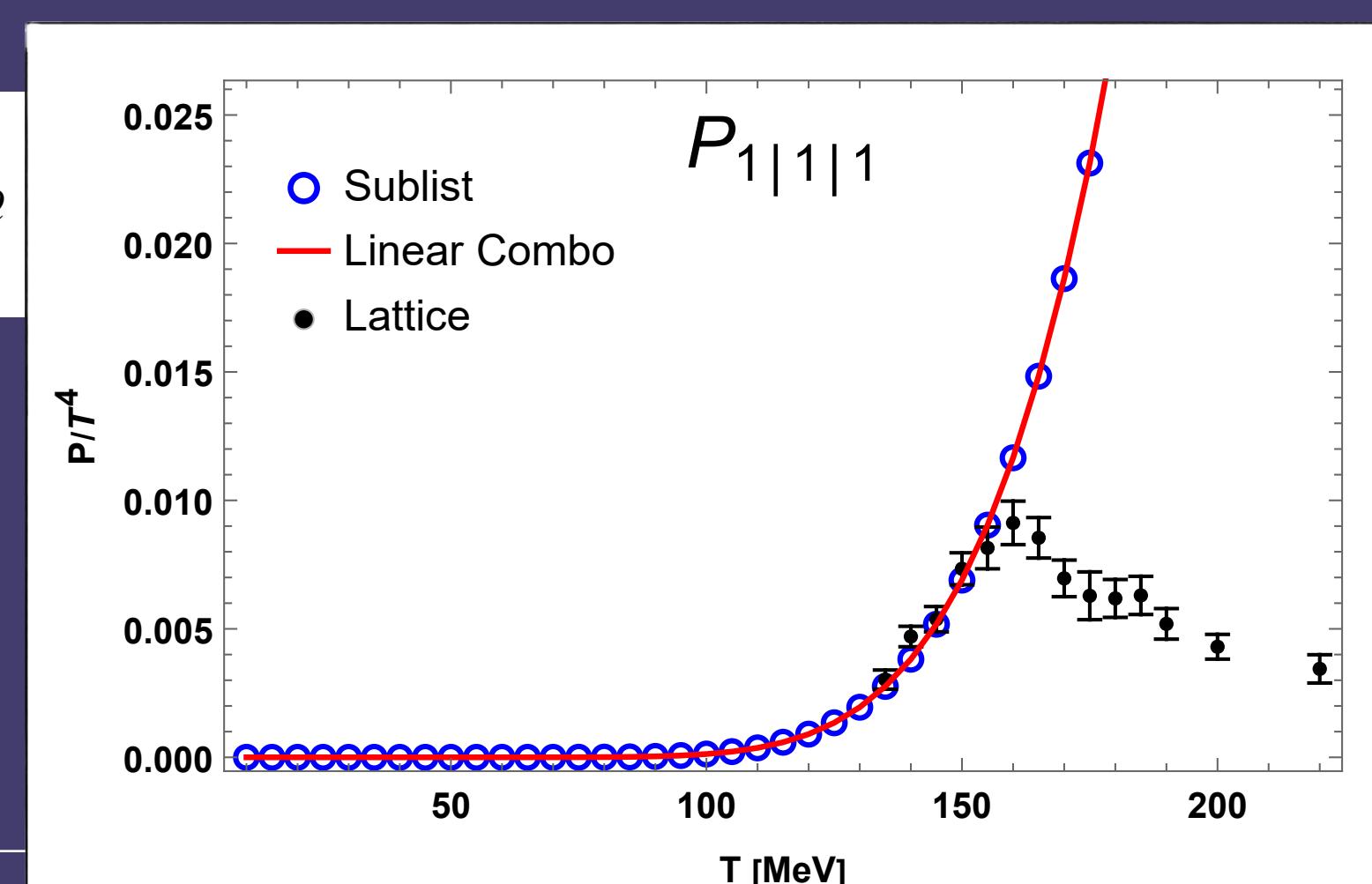
Baryons ($|S|=1$)

$$P_{1|1|-1} = \frac{2}{3}\chi_{13}^{SQ} - \frac{1}{6}\chi_2^S - \frac{1}{2}\chi_{22}^{SQ} - \frac{1}{6}\chi_{31}^{SQ} + \frac{1}{6}\chi_4^S - \frac{1}{3}\chi_{11}^{BS} - \chi_{112}^{BSQ} + \frac{1}{3}\chi_{13}^{BS} + \frac{1}{6}\chi_2^B - \frac{1}{6}\chi_4^B$$



$$P_{1|1|0} = \frac{1}{24}\chi_2^S - \frac{1}{2}\chi_{22}^{SQ} + \frac{1}{2}\chi_{31}^{SQ} - \frac{1}{24}\chi_4^S + \frac{1}{2}\chi_{112}^{BSQ} + \frac{1}{4}\chi_{13}^{BS} - \frac{1}{8}\chi_2^B - \frac{1}{2}\chi_{211}^{BSQ} - \frac{5}{4}\chi_{31}^{BS} + \frac{1}{8}\chi_4^B$$

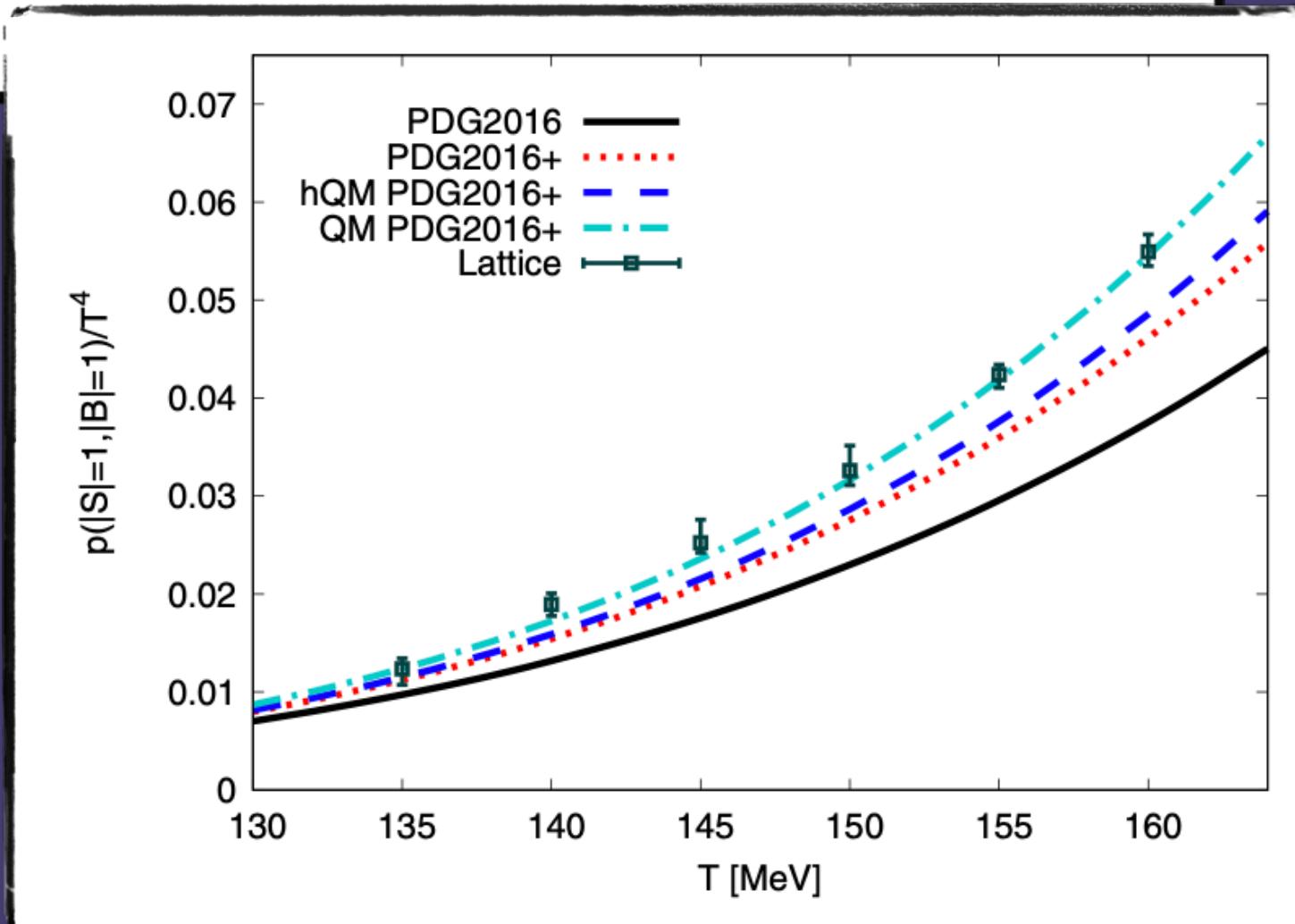
$$P_{1|1|1} = -\frac{1}{4}\chi_{13}^{SQ} + \frac{1}{4}\chi_{22}^{SQ} - \frac{1}{4}\chi_{112}^{BSQ} + \frac{1}{4}\chi_{121}^{BSQ}$$



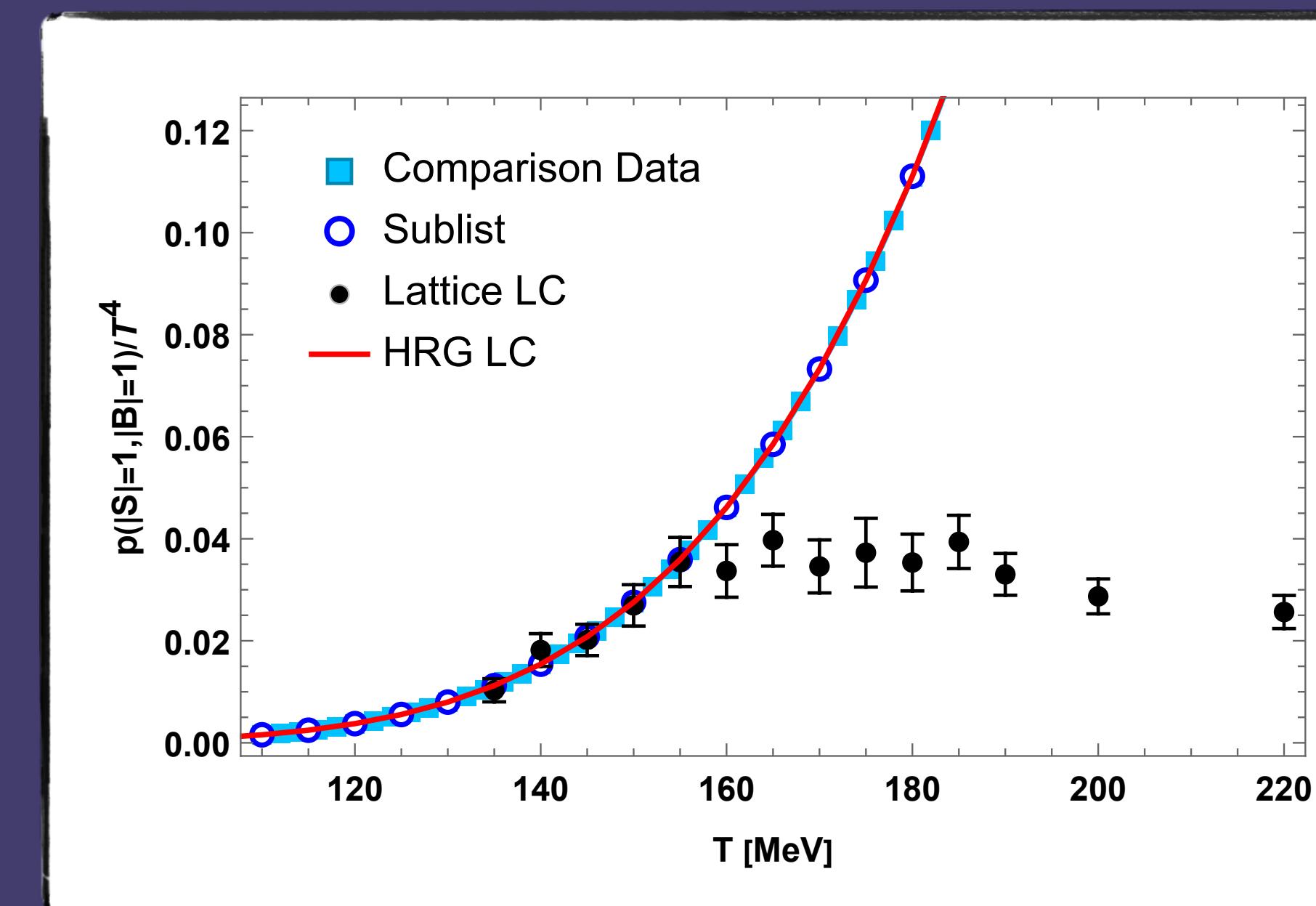
Baryons ($|S|=1$) Comparison Plots

Full Pressure (2D)

$$\begin{aligned}
 P(\hat{\mu}_B, \hat{\mu}_S) = & P_{00}^{BS} + P_{10}^{BS} \cosh(\hat{\mu}_B) + P_{01}^{BS} \cosh(-\hat{\mu}_S) \\
 & + P_{11}^{BS} \cosh(\hat{\mu}_B - \hat{\mu}_S) + P_{12}^{BS} \cosh(\hat{\mu}_B - 2\hat{\mu}_S) \\
 & \cancel{+ P_{13}^{BS} \cosh(\hat{\mu}_B - 3\hat{\mu}_S)},
 \end{aligned}$$



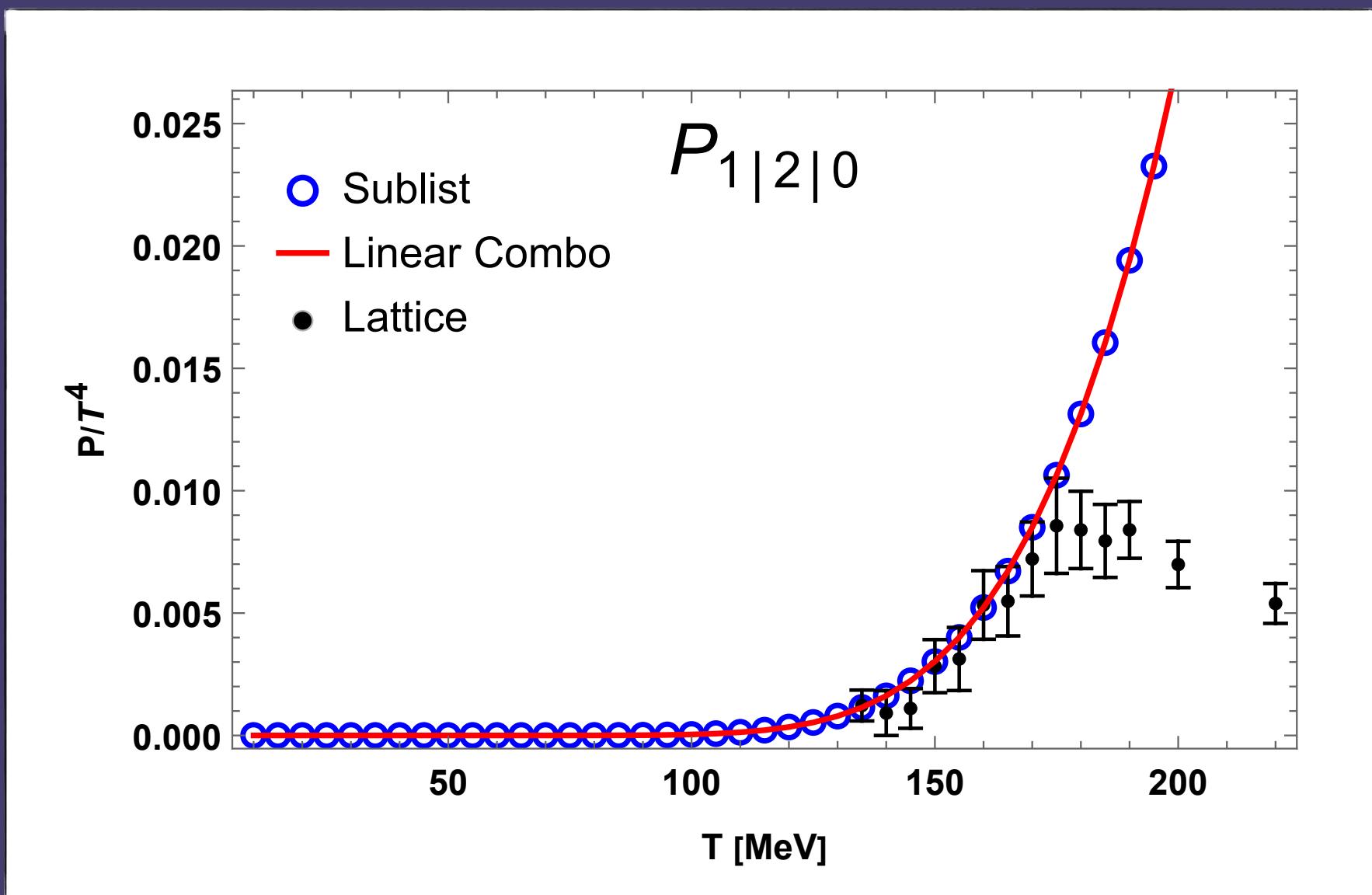
Comparing our Linear Combinations (summed over Q) to 2D version
with PDG2016+ data



$$\begin{aligned}
 |\mathbf{B}| &= 1 \\
 |\mathbf{S}| &= 1 \\
 Q &= -1, 0, 1
 \end{aligned}$$

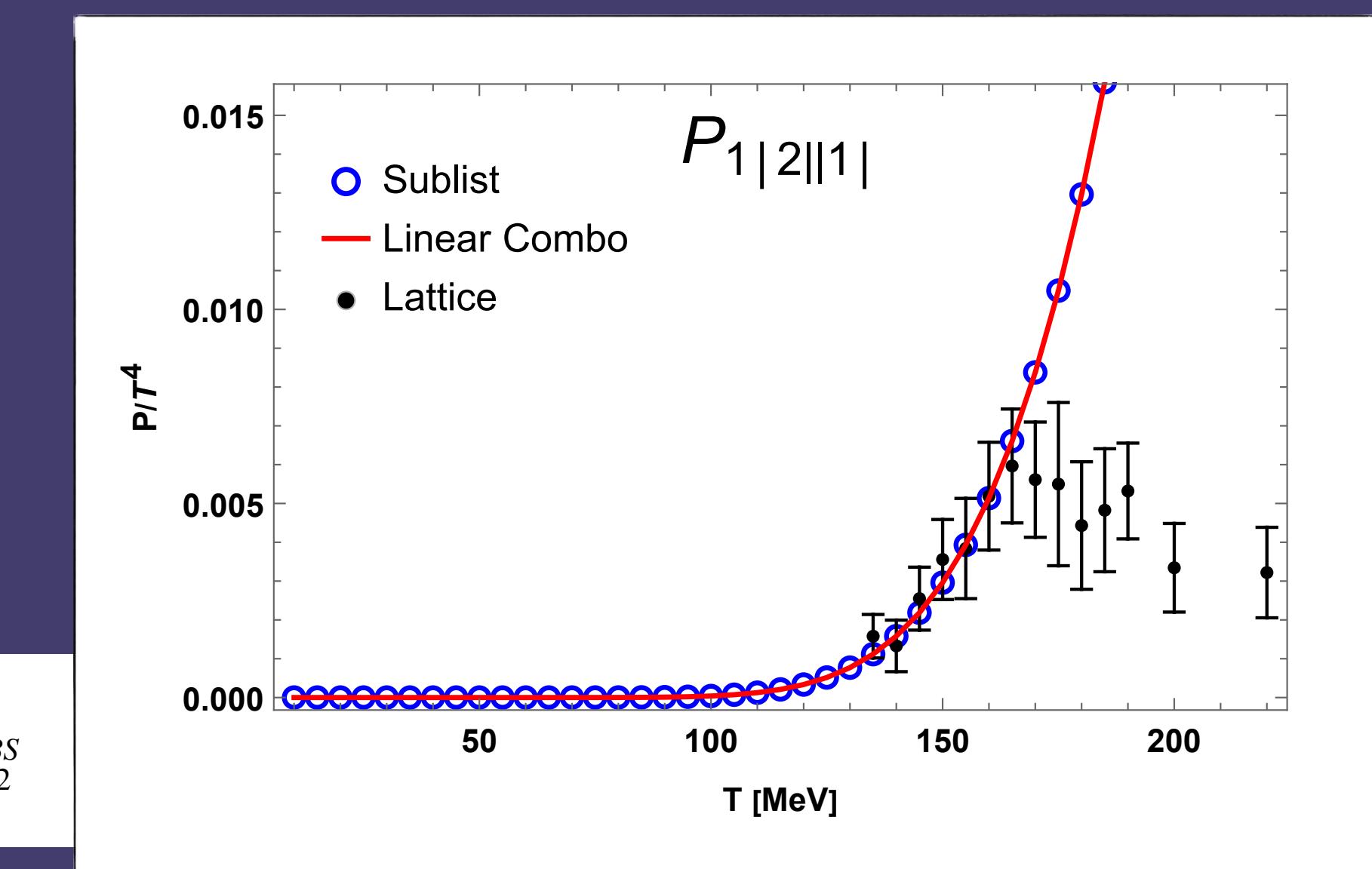
Partial pressures for various particle lists compared to Lattice

Baryons ($|S|=2$)



$$P_{1|2|0} = \frac{1}{6}\chi_{13}^{SQ} - \frac{1}{36}\chi_2^S - \frac{1}{6}\chi_{31}^{SQ} + \frac{1}{36}\chi_4^S + \frac{1}{12}\chi_2^B + \frac{1}{3}\chi_{22}^{BS} + \frac{1}{3}\chi_{31}^{BS} - \frac{1}{12}\chi_4^B$$

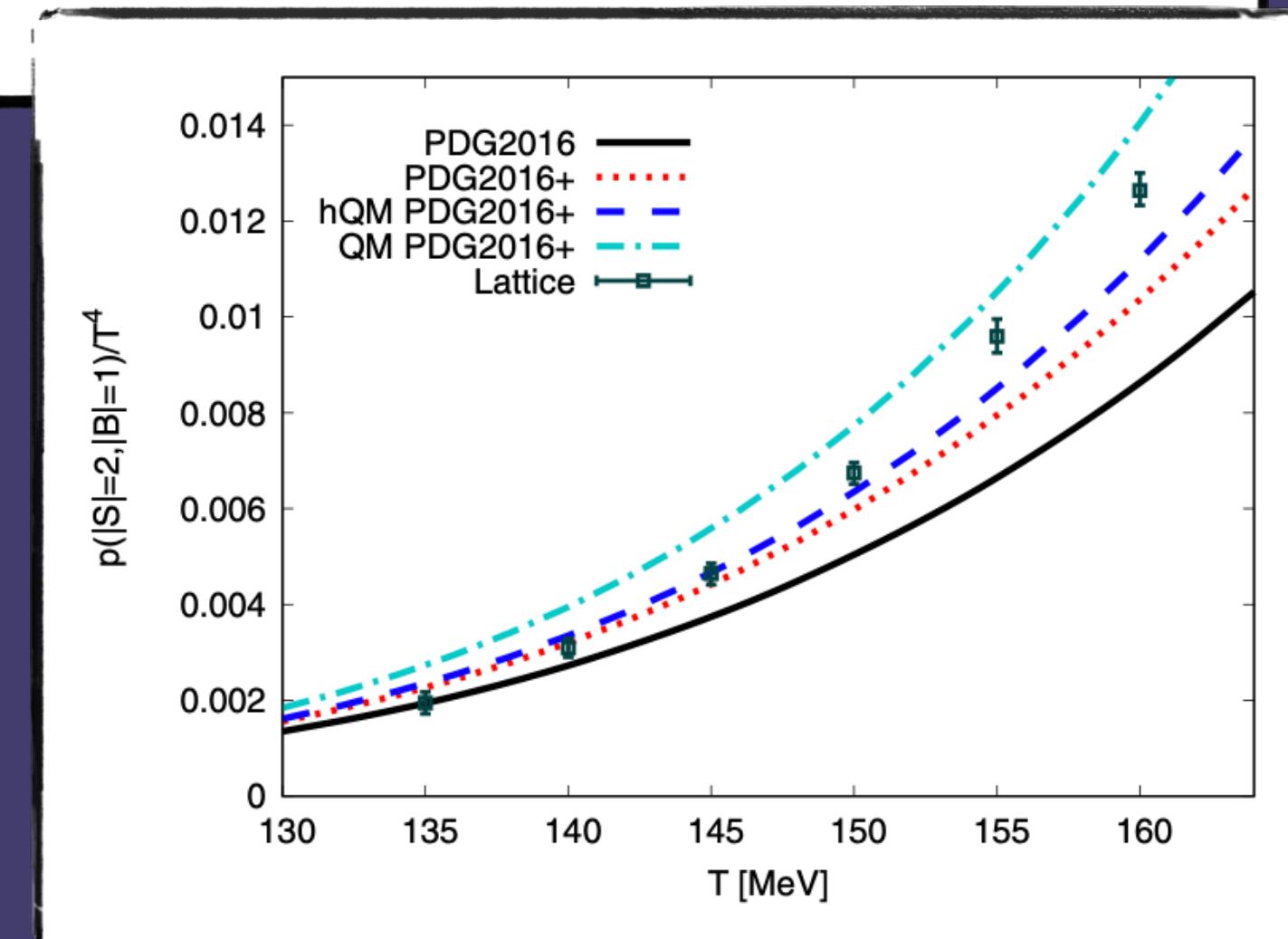
$$P_{1|2||1|} = -\frac{1}{4}\chi_{13}^{SQ} + \frac{1}{9}\chi_2^S + \frac{1}{4}\chi_{22}^{SQ} - \frac{1}{9}\chi_4^S + \frac{1}{3}\chi_{11}^{BS} + \frac{1}{4}\chi_{112}^{BSQ} - \frac{1}{4}\chi_{121}^{BSQ} - \frac{1}{6}\chi_{13}^{BS} + \frac{1}{6}\chi_{22}^{BS}$$



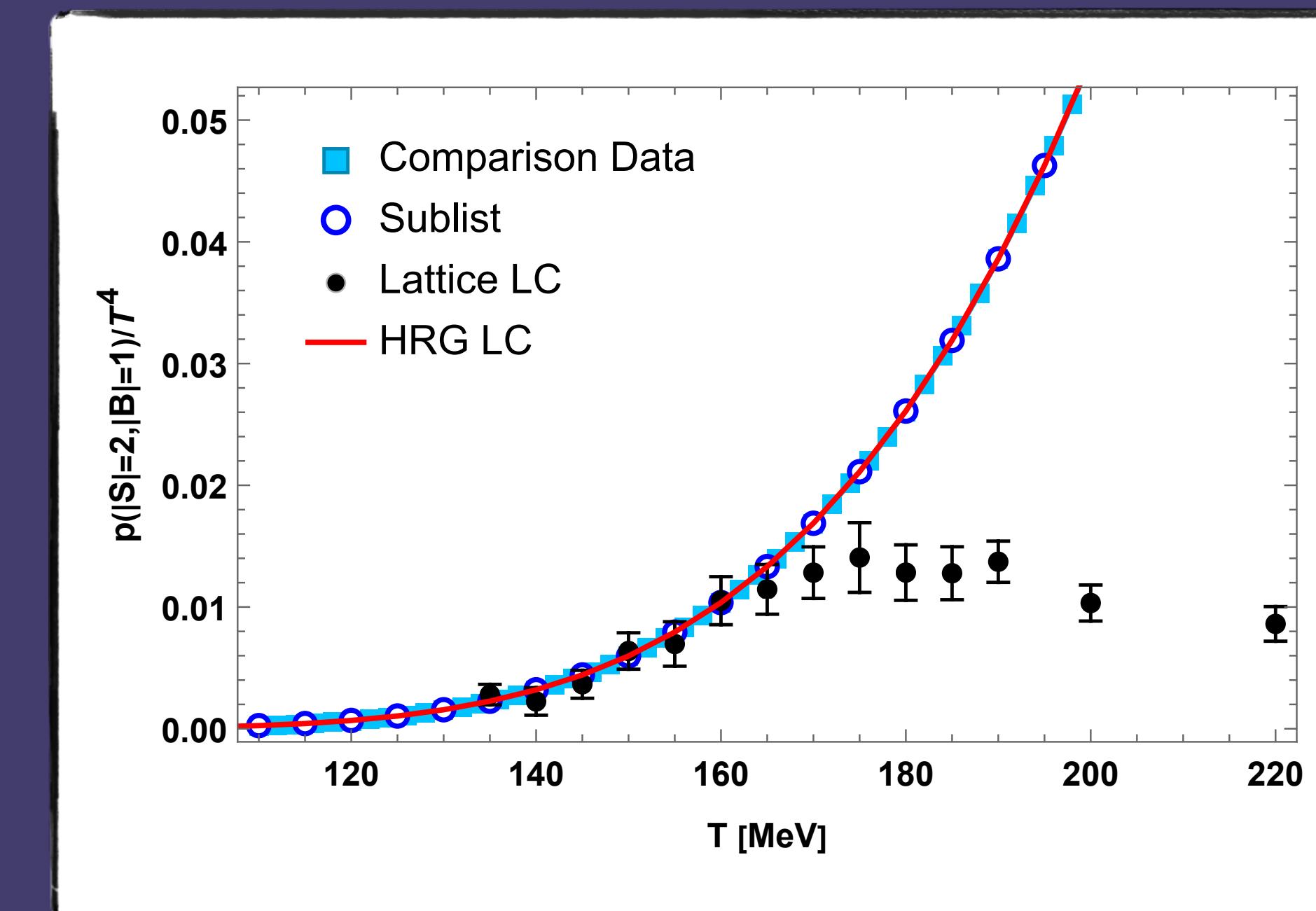
Baryons ($|S|=2$) Comparison Plots

Full Pressure (2D)

$$\begin{aligned}
 P(\hat{\mu}_B, \hat{\mu}_S) = & P_{00}^{BS} + P_{10}^{BS} \cosh(\hat{\mu}_B) + P_{01}^{BS} \cosh(-\hat{\mu}_S) \\
 & + P_{11}^{BS} \cosh(\hat{\mu}_B - \hat{\mu}_S) + P_{12}^{BS} \cosh(\hat{\mu}_B - 2\hat{\mu}_S) \\
 & + P_{13}^{BS} \cosh(\hat{\mu}_B - 3\hat{\mu}_S),
 \end{aligned}$$



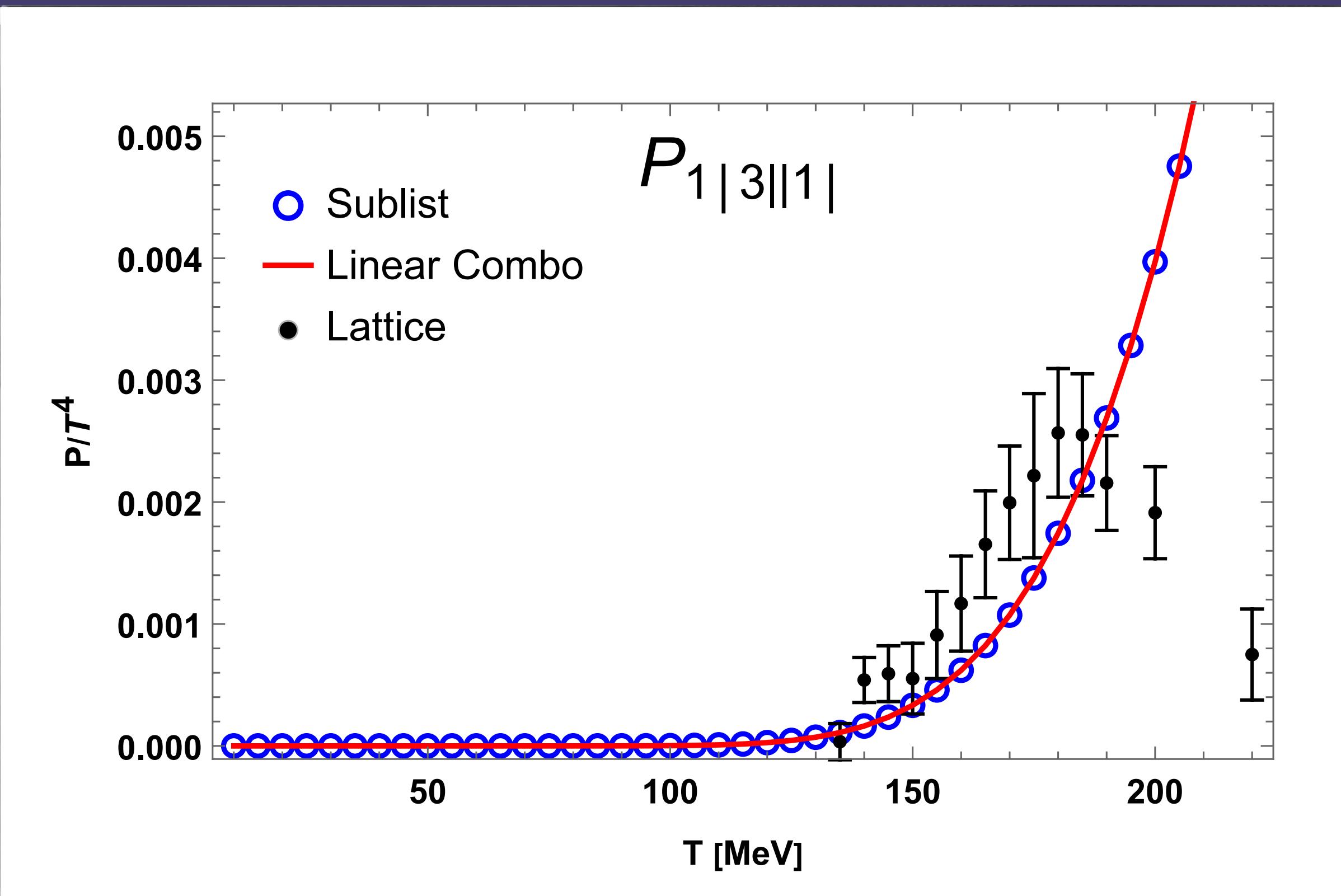
Comparing our Linear Combinations (summed over Q) to 2D version with PDG2016+ data



$|B| = 1$
 $|S| = 2$
 $|Q| = 0, 1$

Partial pressures for various particle lists compared to Lattice

Baryons ($|S|=3$)

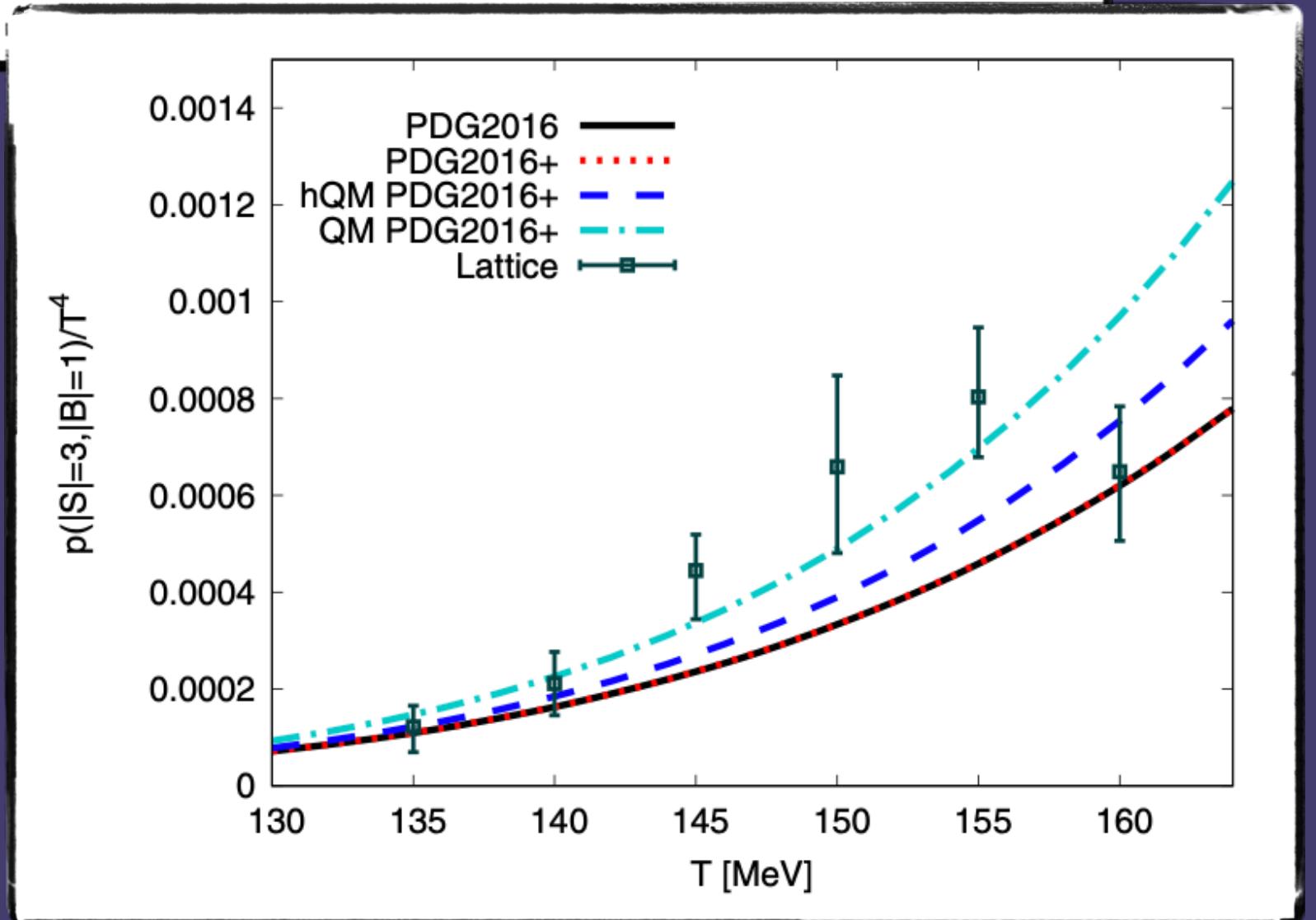


$$P_{1|3||1|} = \frac{\pi^2}{18 - 27\pi^2} \chi_2^S + \frac{\pi^2}{9(-2 + 3\pi^2)} \chi_4^S + \frac{2 - \pi^2}{-6 + 9\pi^2} \chi_{11}^{BS} + \frac{2 + \pi^2}{6(-2 + 3\pi^2)} \chi_{13}^{BS} - \frac{-6 + \pi^2}{6(-2 + 3\pi^2)} \chi_{22}^{BS}$$

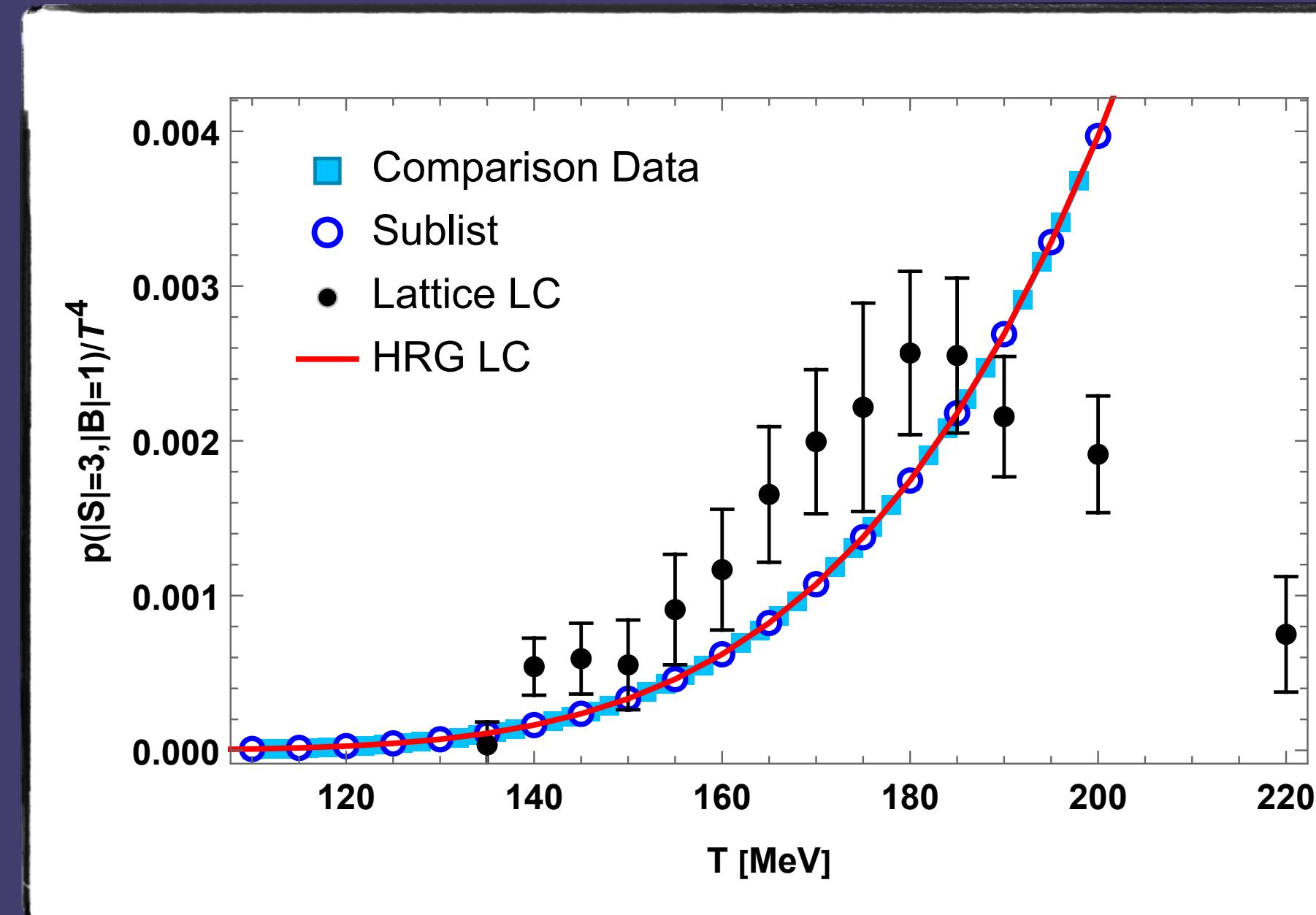
Baryons ($|S|=3$) Comparison Plots

Full Pressure (2D)

$$\begin{aligned}
 P(\hat{\mu}_B, \hat{\mu}_S) = & P_{00}^{BS} + P_{10}^{BS} \cosh(\hat{\mu}_B) + P_{01}^{BS} \cosh(-\hat{\mu}_S) \\
 & + P_{11}^{BS} \cosh(\hat{\mu}_B - \hat{\mu}_S) + P_{12}^{BS} \cosh(\hat{\mu}_B - 2\hat{\mu}_S) \\
 & + P_{13}^{BS} \cosh(\hat{\mu}_B - 3\hat{\mu}_S),
 \end{aligned}$$



Comparing our Linear Combinations (summed over Q) to 2D version
with PDG2016+ data



$|B| = 1$
 $|S| = 3$
 $|Q| = 1$

Partial pressures for various particle lists compared to Lattice

In Conclusion

The HRG model helps us understand the contributions to the Lattice QCD pressure from different hadron families.

By solving a system of equations on the full HRG pressure, we find partial pressures as linear combinations of HRG susceptibilities.

This allows us to find the partial pressures from first principles using susceptibilities calculated using Lattice QCD.

Finally, by requiring the correct asymptotic (the SB limit) for these partial pressures we observe non-monotonic behavior which can model deconfinement.

Recommended Reading: *The Deconfinement Phase Transition: Theory Meets Experiment* by Claudia Ratti and Rene Bellwied

Extra Info

With 21 susceptibilities we have 203490 different ways to combine 13 susceptibilities.

Partial pressures with a S.B. limit of zero included by construction.

With 21 susceptibilities we have 116280 different ways to combine 14 susceptibilities.

Only a subset of these lead to valid solutions for each partial pressure

{15099, 11524, 13372, 12440, 48242, 20110, 22373, 19571, 20566, 19796, 24369, 24583, 26794}

Ensuring that pressure remains positive reduces the number of valid solutions

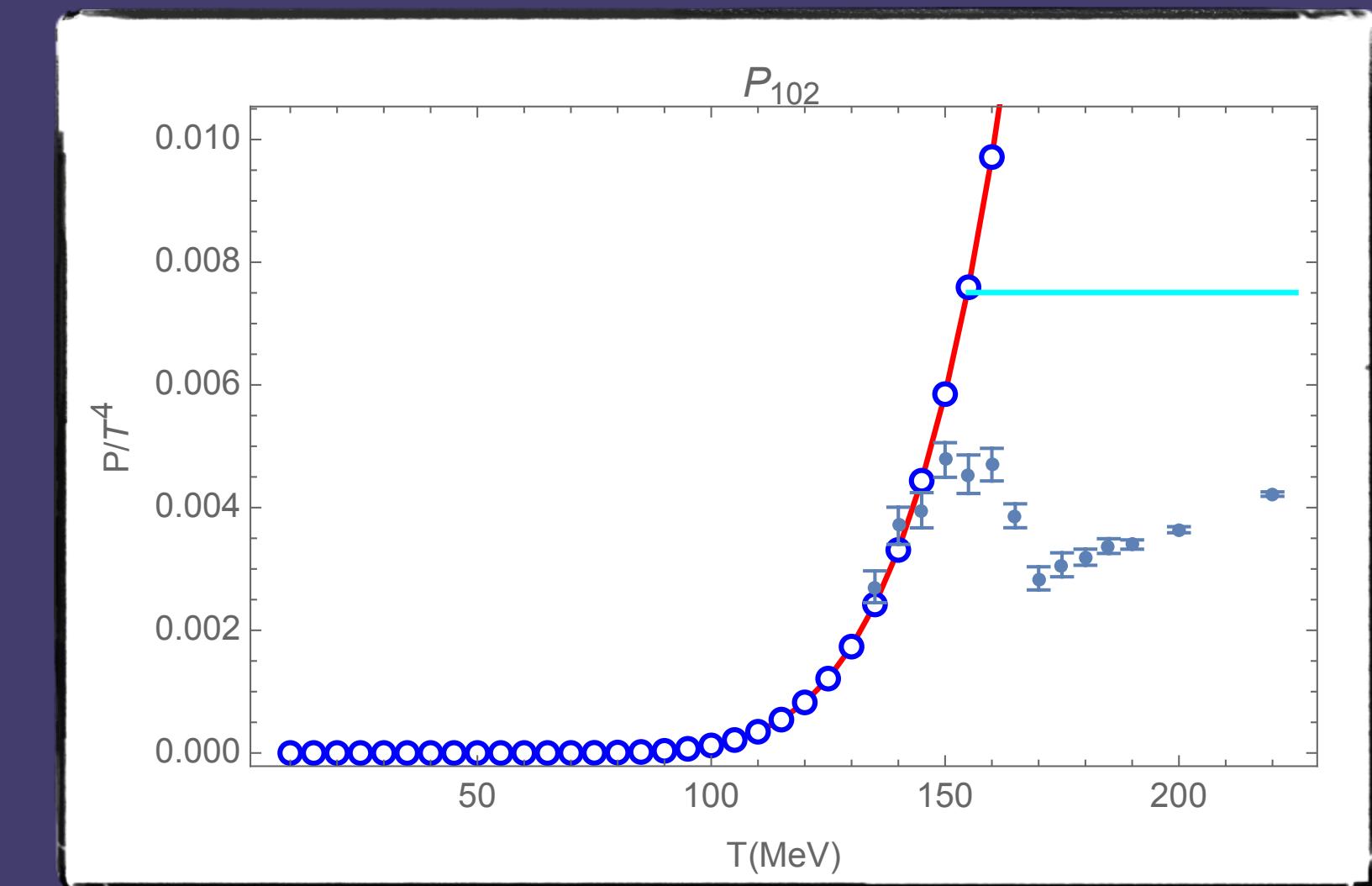
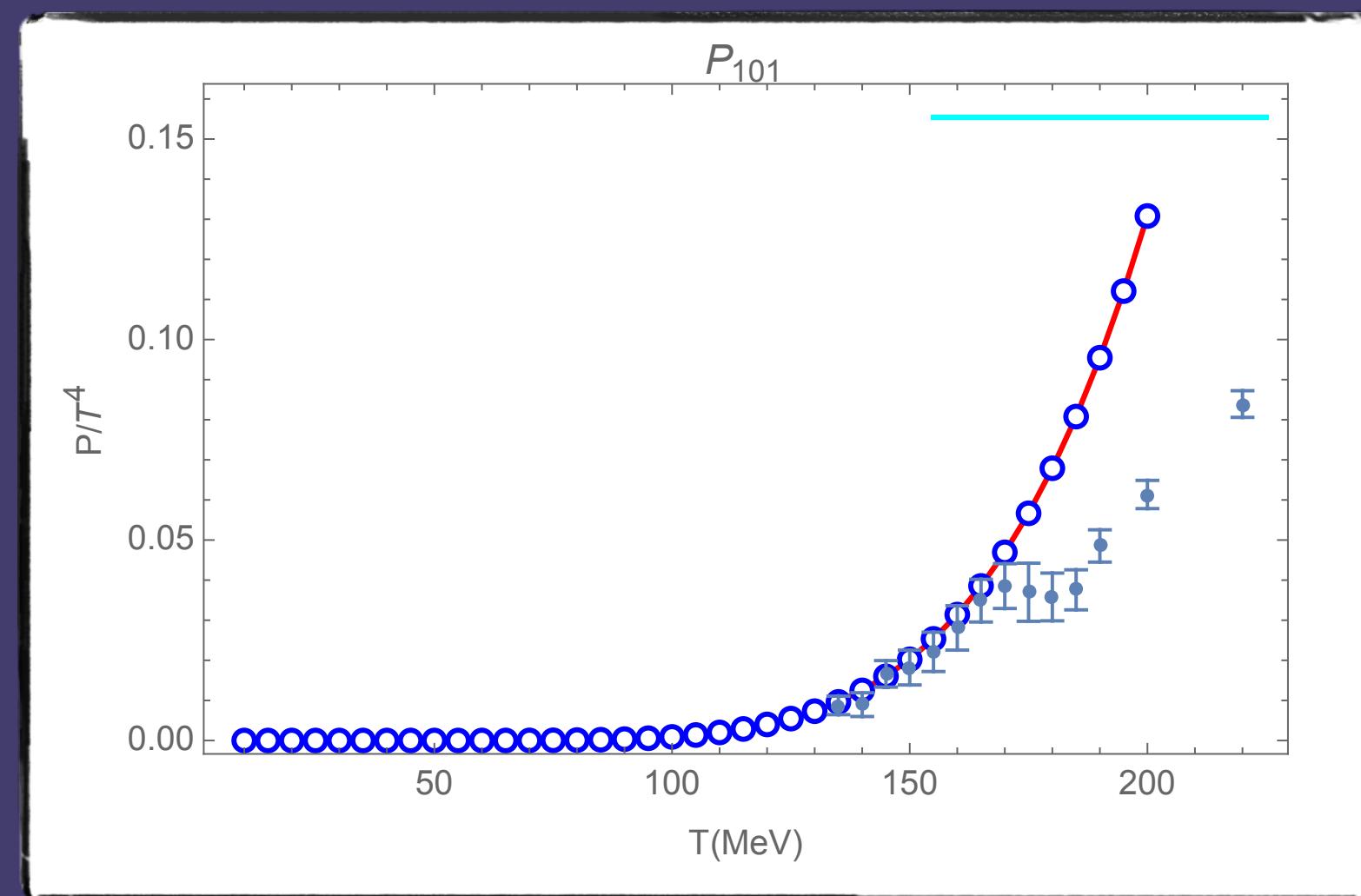
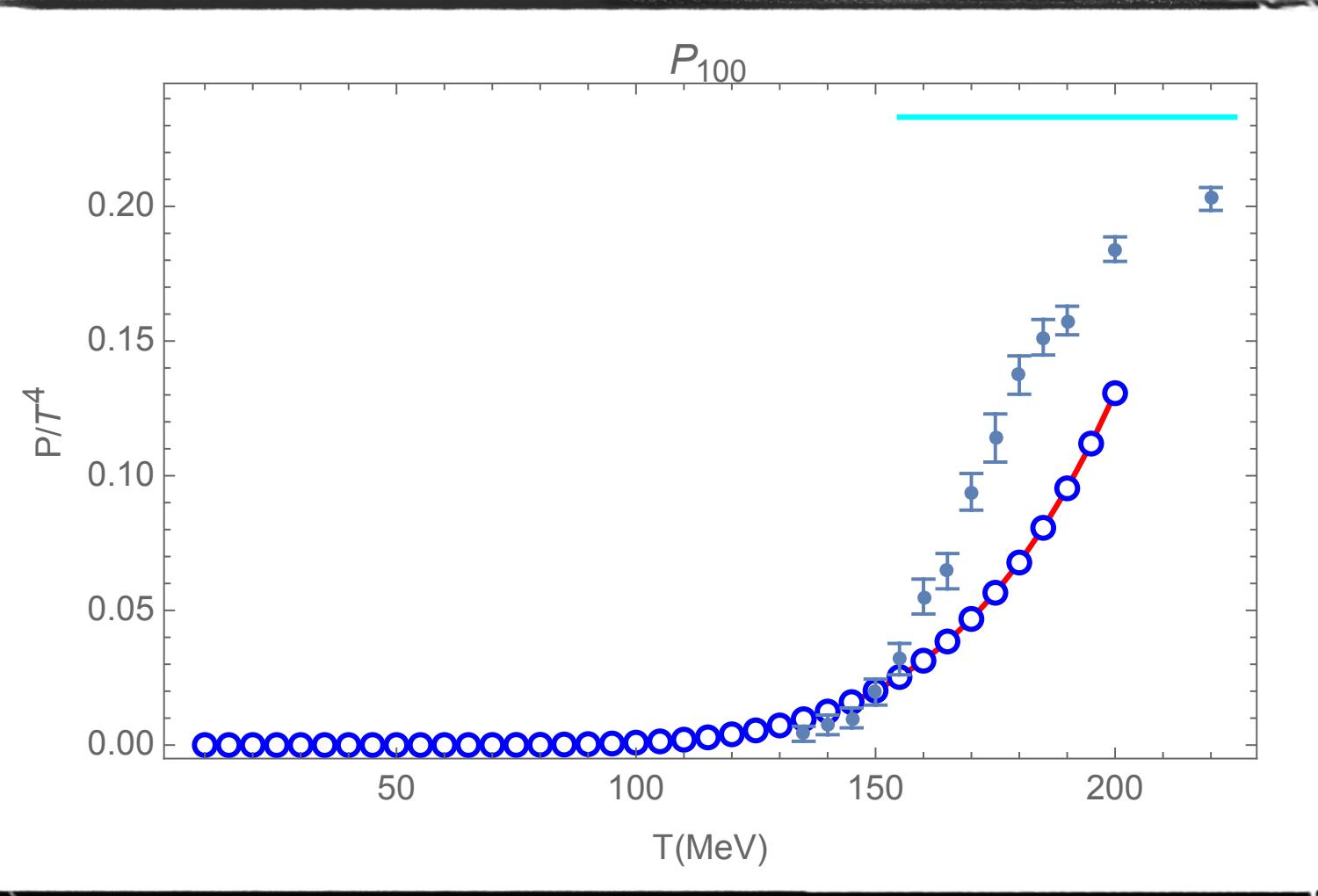
{14895, 4220, 8073, 6511, 3538, 15450, 14458, 13233, 14902, 4335, 8815, 16207, 11108}

Grouping duplicates (exact same behavior for partial pressures on lattice)

{631, 773, 764, 1651, 68, 484, 186, 312, 221, 258, 105, 264, 76}

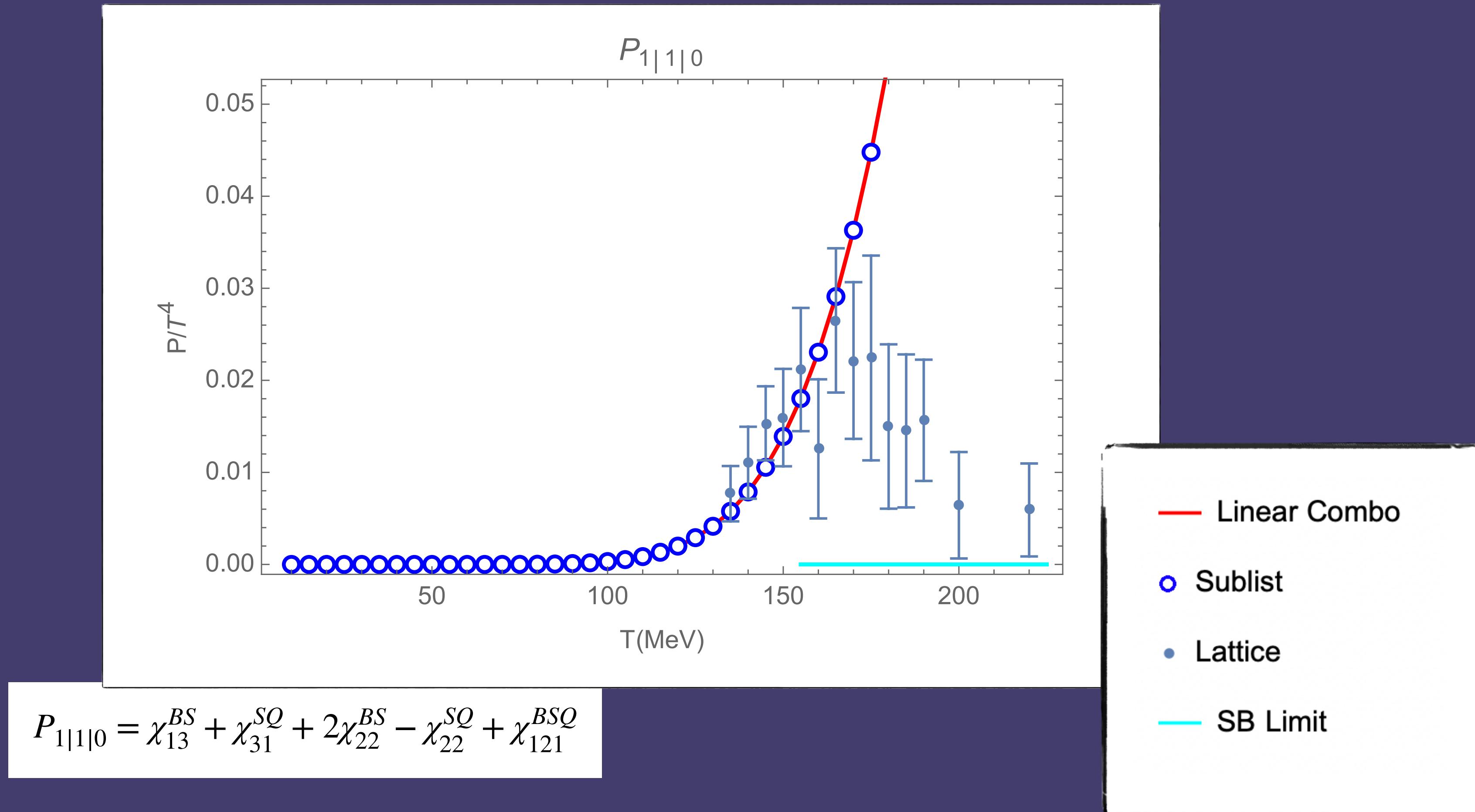
Partial Pressure Plots w/ Lattice

$$P_{Partial} = \sum_s c_s \chi_s^{LAT}$$



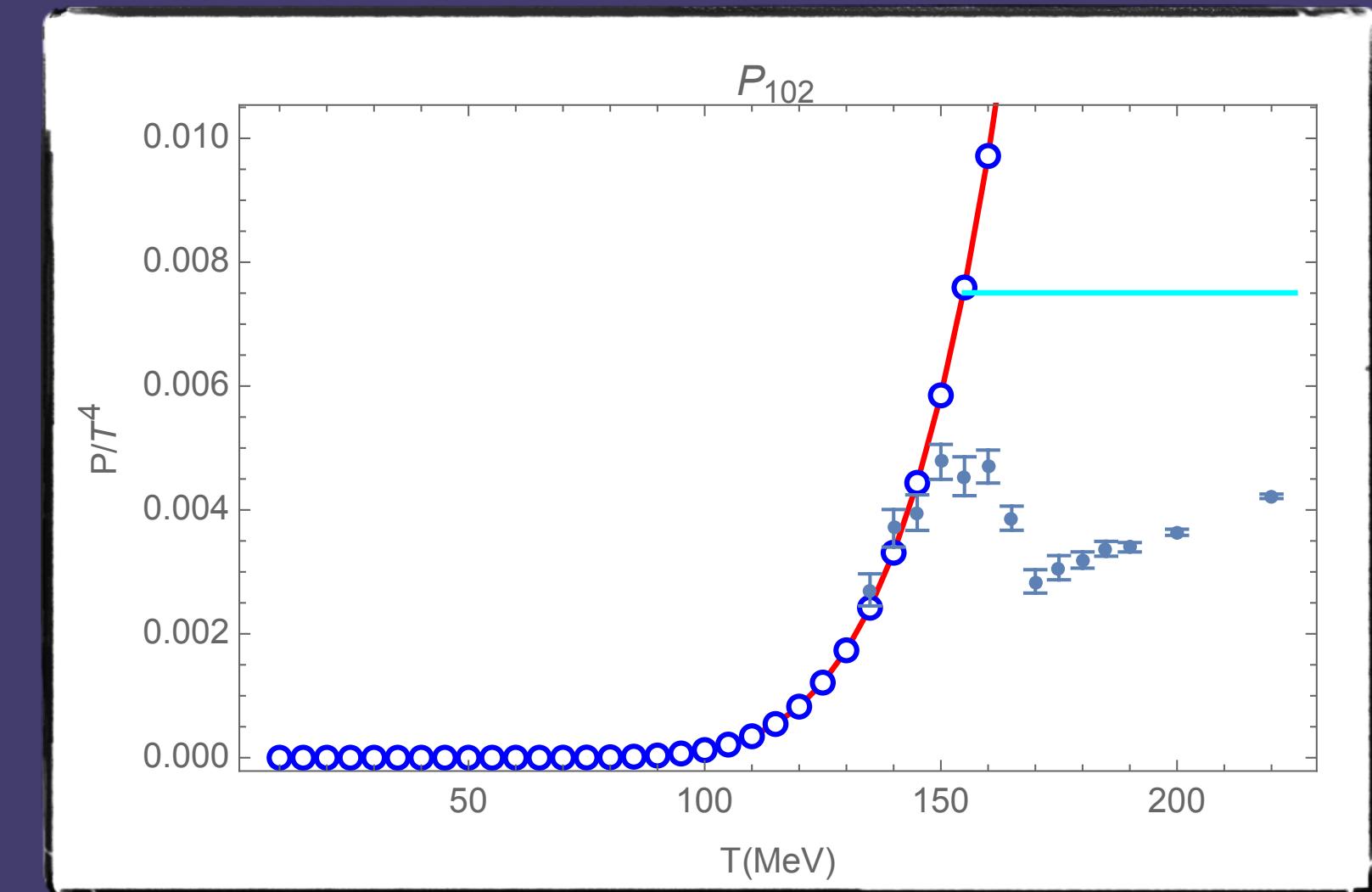
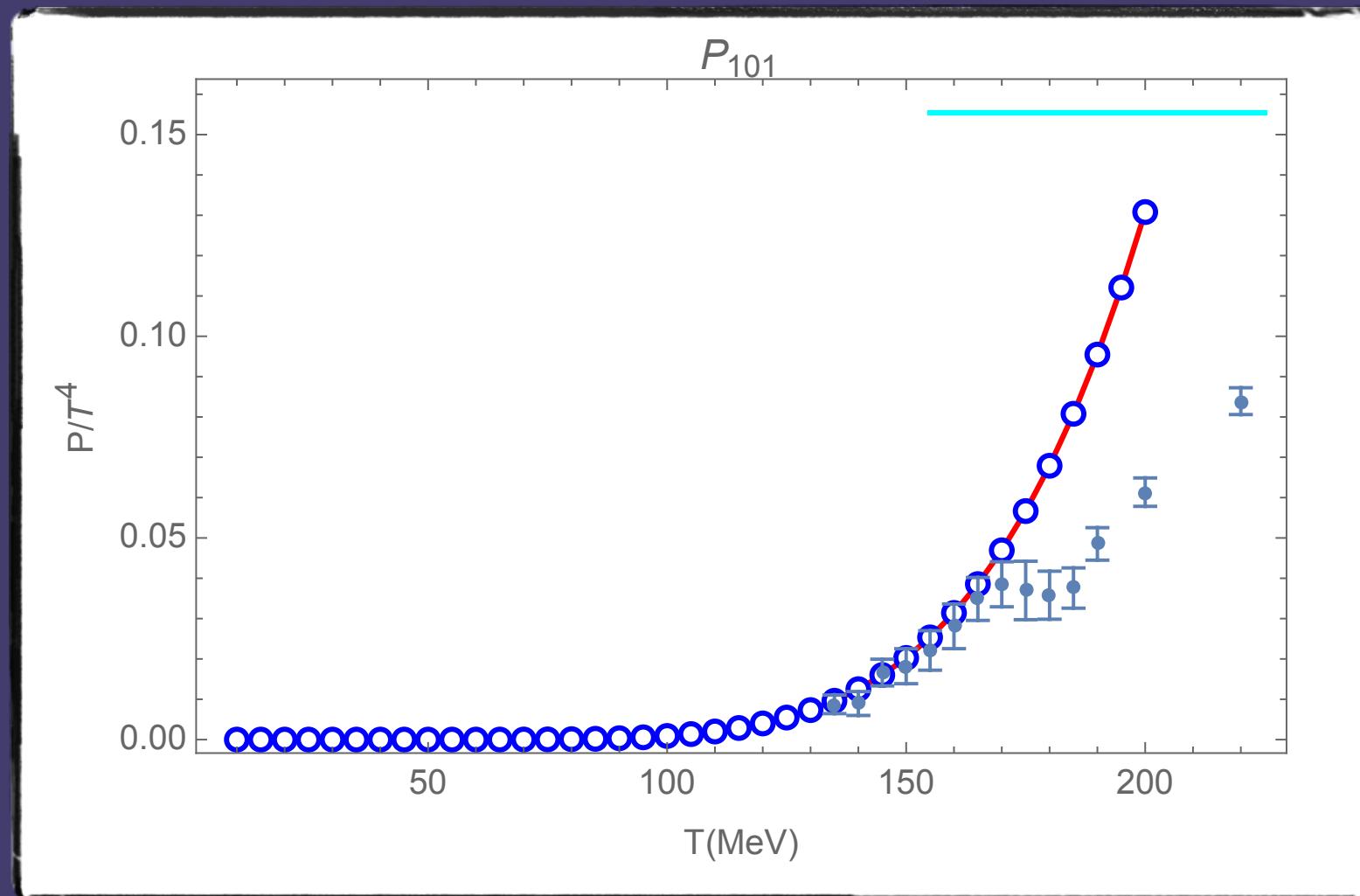
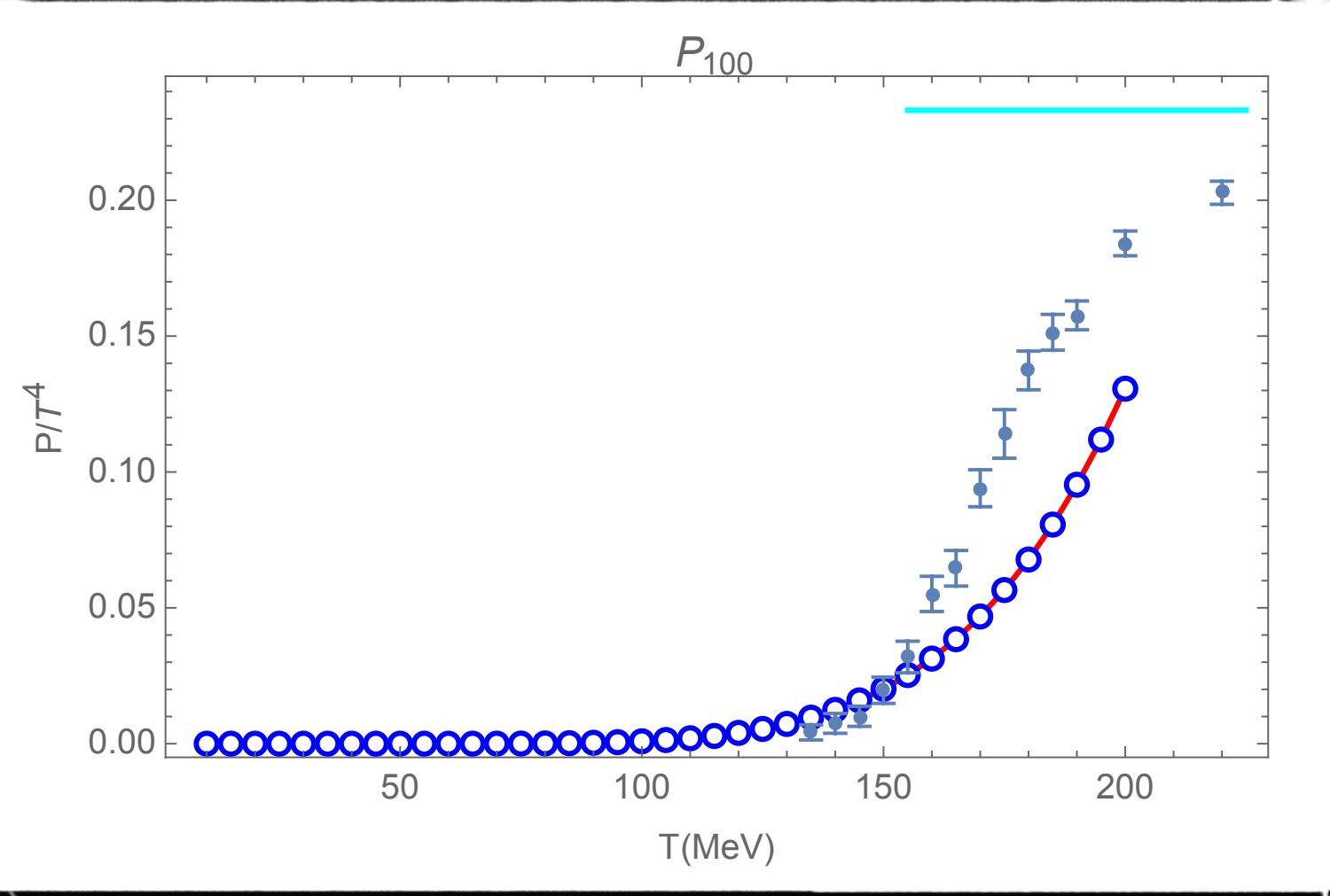
- Linear Combo
- Sublist
- Lattice
- SB Limit

Rescaling to S.B. Limit = 0



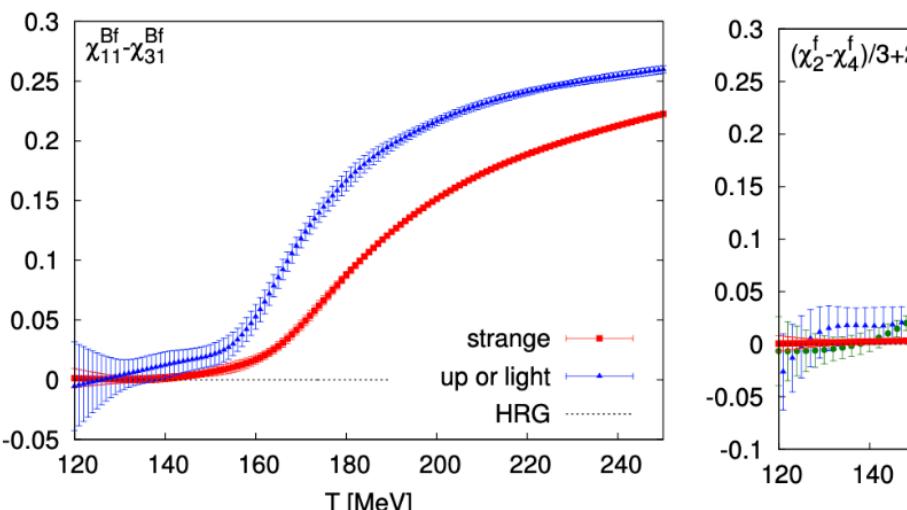
Partial Pressure Plots w/ Lattice

$$P_{Partial} = \sum_s c_s \chi_s^{LAT}$$

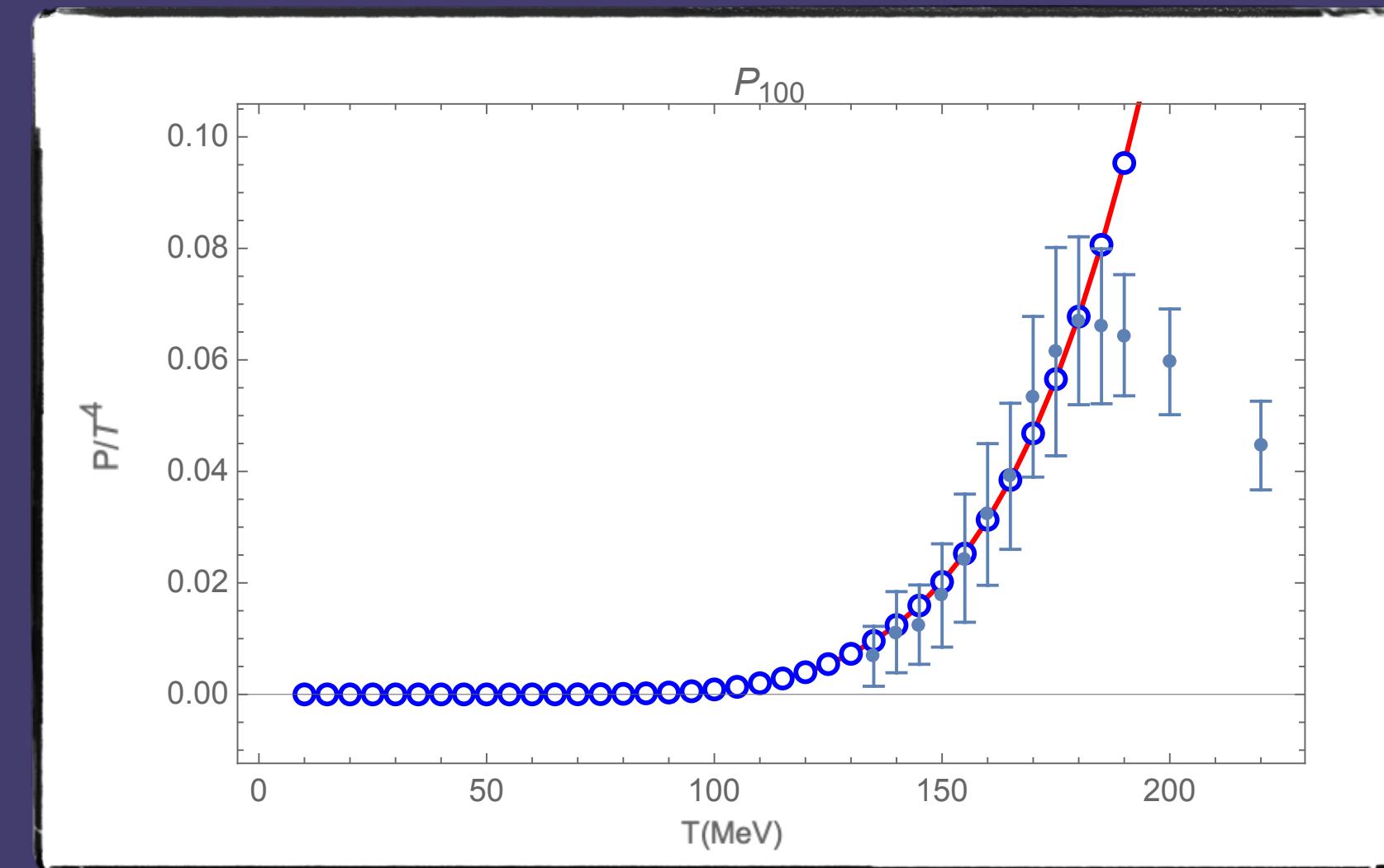
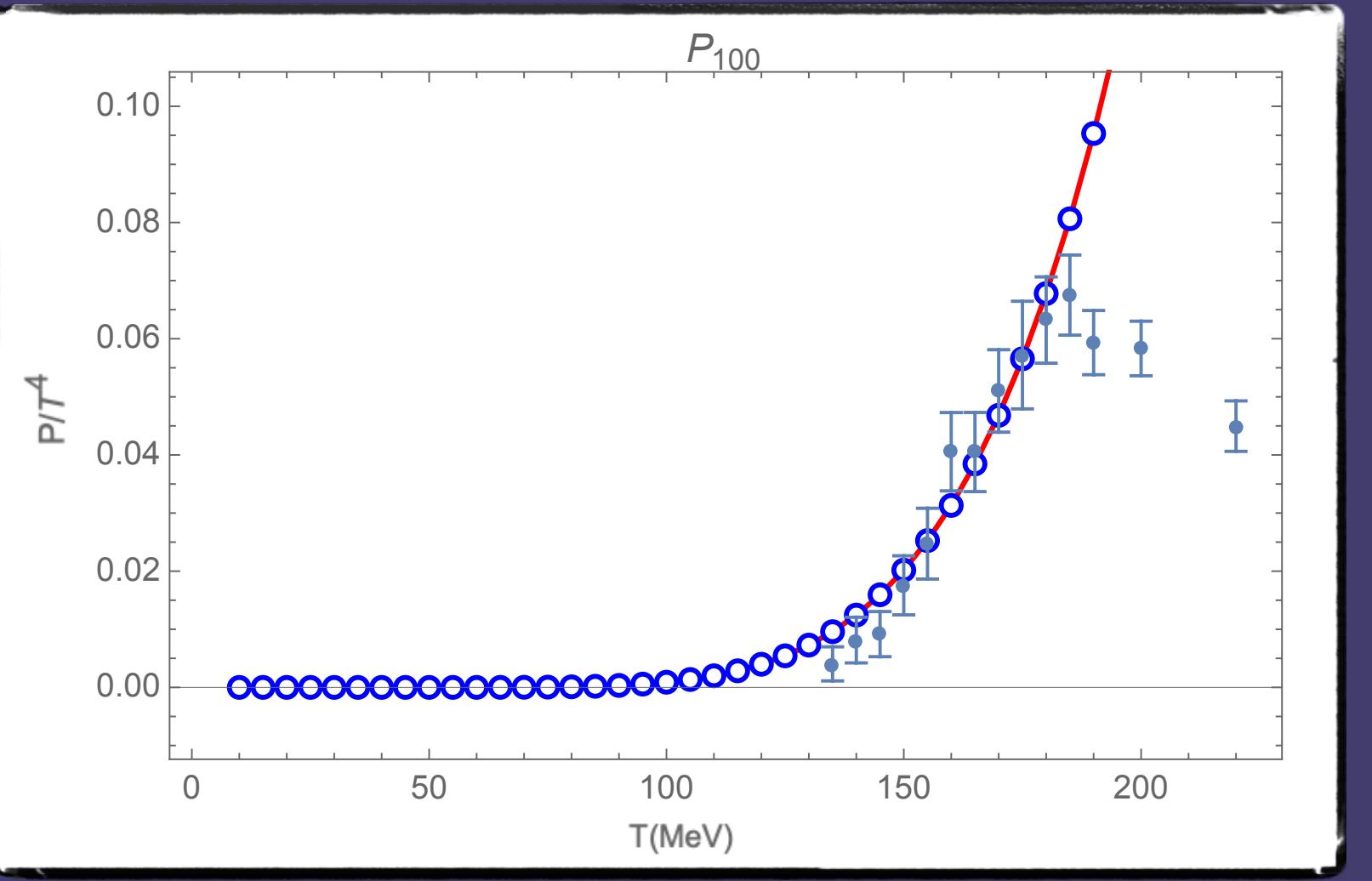
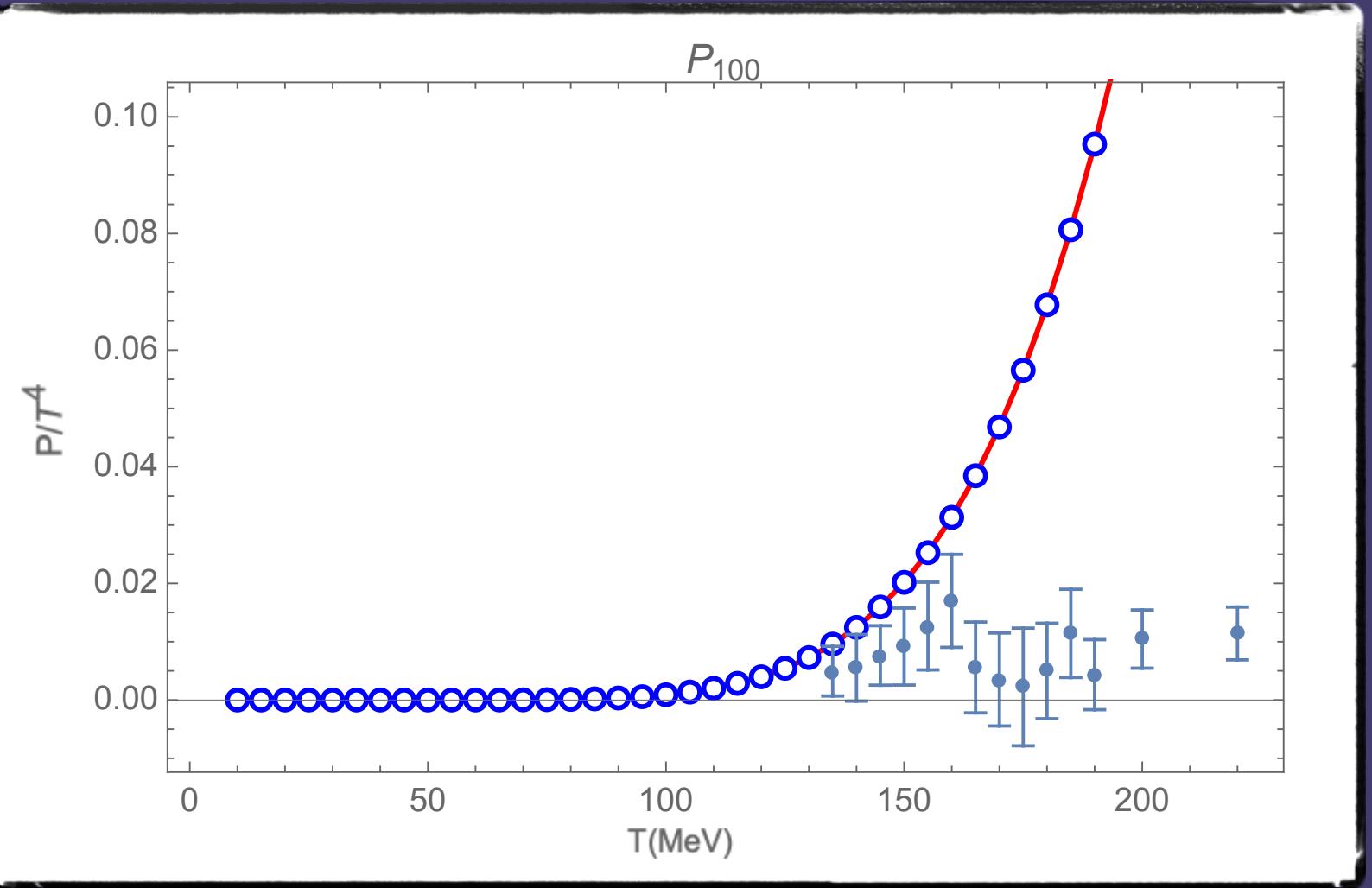


<ul style="list-style-type: none"> — Linear Combo ○ Sublist ● Lattice — SB Limit

Rescaling Lattice (via Old Method)



v1 light



$$P_{Partial} = \sum_s c_s \chi_s^{LAT} + c_i v_i^f$$

- Linear Combo
- Sublist
- Lattice
- SB Limit